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Polynomial and Quadratic Expressions, Equations, and Functions

[1] Each lesson is ONE day, and ONE day is considered a 45-minute period.

Algebra I • Module 4

Polynomial and Quadratic Expressions, Equations, and Functions

OVERVIEW

By the end of middle school, students are familiar with linear equations in one variable (**6.EE.B.5**, **6.EE.B.6**, **6.EE.B.7**) and have applied graphical and algebraic methods to analyze and manipulate equations in two variables (**7.EE.A.2**). They used expressions and equations to solve real-life problems (**7.EE.B.4**). They have experience with square and cube roots, irrational numbers (**8.NS.A.1**), and expressions with integer exponents (**8.EE.A.1**).

In Algebra I, students have been analyzing the process of solving equations and developing fluency in writing, interpreting, and translating among various forms of linear equations (Module 1) and linear and exponential functions (Module 3). These experiences, combined with modeling with data (Module 2), set the stage for Module 4. Here, students continue to interpret expressions, create equations, rewrite equations and functions in different but equivalent forms, and graph and interpret functions using polynomial functions—more specifically quadratic functions as well as square root and cube root functions.

Topic A introduces polynomial expressions. In Module 1, students learned the definition of a polynomial and how to add, subtract, and multiply polynomials. Here, their work with multiplication is extended and connected to factoring polynomial expressions and solving basic polynomial equations (**A-APR.A.1**, **A-REI.D.11**). They analyze, interpret, and use the structure of polynomial expressions to multiply and factor polynomial expressions (**A-SSE.A.2**). They understand factoring as the reverse process of multiplication. In this topic, students develop the factoring skills needed to solve quadratic equations and simple polynomial equations by using the zero product property (**A-SSE.B.3a**). Students transform quadratic expressions from standard form, $ax^2 + bx + c$, to factored form, $f(x) = a(x - n)(x - m)$, and then solve equations involving those expressions. They identify the solutions of the equation as the zeros of the related function. Students apply symmetry to create and interpret graphs of quadratic functions (**F-IF.B.4**, **F-IF.C.7a**). They use average rate of change on an interval to determine where the function is increasing or decreasing (**F-IF.B.6**). Using area models, students explore strategies for factoring more complicated quadratic expressions, including the product-sum method and rectangular arrays. They create one- and two-variable equations from tables, graphs, and contexts and use them to solve contextual problems represented by the quadratic function (**A-CED.A.1**, **A-CED.A.2**). Students then relate the domain and range for the function to its graph and the context (**F-IF.B.5**).

Students apply their experiences from Topic A as they transform quadratic functions from standard form to vertex form, $f(x) = a(x - h)^2 + k$, in Topic B. The strategy known as *completing the square* is used to solve quadratic equations when the quadratic expression cannot be factored (**A-SSE.B.3b**). Students recognize that this form reveals specific features of quadratic functions and their graphs, namely the *minimum* or *maximum of the function* (i.e., the vertex of the graph) and the line of symmetry of the graph (**A-APR.B.3**, **F-IF.B.4**,

F-IF.C.7a). Students derive the quadratic formula by completing the square for a general quadratic equation in standard form, $y = ax^2 + bx + c$, and use it to determine the nature and number of solutions for equations when y equals zero (**A-SSE.A.2**, **A-REI.B.4**). For quadratics with irrational roots, students use the quadratic formula and explore the properties of irrational numbers (**N-RN.B.3**). With the added technique of completing the square in their toolboxes, students come to see the structure of the equations in their various forms as useful for gaining insight into the features of the graphs of equations (**A-SSE.B.3**). Students study business applications of quadratic functions as they create quadratic equations and graphs from tables and contexts and then use them to solve problems involving profit, loss, revenue, cost, etc. (**A-CED.A.1**, **A-CED.A.2**, **F-IF.B.6**, **F-IF.C.8a**). In addition to applications in business, students solve physics-based problems involving objects in motion. In doing so, students also interpret expressions and parts of expressions in context and recognize when a single entity of an expression is dependent or independent of a given quantity (**A-SSE.A.1**).

In Topic C, students explore the families of functions that are related to the parent functions, specifically for quadratic ($f(x) = x^2$), square root ($f(x) = \sqrt{x}$), and cube root ($f(x) = \sqrt[3]{x}$), to perform horizontal and vertical translations as well as shrinking and stretching (**F-IF.C.7b**, **F-BF.B.3**). They recognize the application of transformations in vertex form for a quadratic function and use it to expand their ability to efficiently sketch graphs of square and cube root functions. Students compare quadratic, square root, or cube root functions in context and represent each in different ways (verbally with a description, numerically in tables, algebraically, or graphically). In the final two lessons, students examine real-world problems of quadratic relationships presented as a data set, a graph, a written relationship, or an equation. They choose the most useful form for writing the function and apply the techniques learned throughout the module to analyze and solve a given problem (**A-CED.A.2**), including calculating and interpreting the rate of change for the function over an interval (**F-IF.B.6**).

Focus Standards

Use properties of rational and irrational numbers.

N-RN.B.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Interpret the structure of expressions.

A-SSE.A.1 Interpret expressions that represent a quantity in terms of its context.★

a. Interpret parts of an expression, such as terms, factors, and coefficients.[2]

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1 + r)^n$ *as the product of* P *and a factor not depending on* P.

[2] The "such as" listed are not the only parts of an expression students are expected to know; others include, but are not limited to, degree of a polynomial, leading coefficient, constant term, and the standard from of a polynomial (descending exponents)(in preparation for Regents Exam).

A-SSE.A.2 Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*[3]

Write expressions in equivalent forms to solve problems.

A-SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★

a. Factor a quadratic expression to reveal the zeros of the function it defines.[4]

b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

Perform arithmetic operations on polynomials.

A-APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Understand the relationship between zeros and factors of polynomials.

A-APR.B.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.[5]

Create equations that describe numbers or relationships.

A-CED.A.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*★[6]

A-CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

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[3] In Algebra I, tasks are limited to numerical expressions and polynomial expressions in one variable. Examples: Recognize that $53^2 - 47^2$ is the difference of squares and see an opportunity to rewrite it in the easier-to-evaluate form $(53 - 47)(53 + 47)$. See an opportunity to rewrite $a^2 + 9a + 14$ as $(a + 7)(a + 2)$. This does not include factoring by grouping and factoring the sum and difference of cubes (in preparation for Regents Exams).
[4] Includes trinomials and leading coefficients other than 1 (in preparation for Regents Exams).
[5] In Algebra I, tasks are limited to quadratic and cubic polynomials, in which linear and quadratic factors are available. For example, find the zeros of $(x - 2)(x^2 - 9)$.
[6] In Algebra I, tasks are limited to linear, quadratic, or exponential equations with integer exponents.

Solve equations and inequalities in one variable.

A-REI.B.4 Solve quadratic equations in one variable.[7]

a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.[8]

Represent and solve equations and inequalities graphically.

A-REI.D.11[9] Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★

Interpret functions that arise in applications in terms of the context.

F-IF.B.4[10] For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

F-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

F-IF.B.6[11] Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

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[7] Solutions may include simplifying radicals (in preparation for Regents Exams).

[8] Tasks do not require students to write solutions for quadratic equations that have roots with nonzero imaginary parts. However, tasks can require that students recognize cases in which a quadratic equation has no real solutions.

[9] In Algebra I, tasks that assess conceptual understanding of the indicated concept may involve any of the function types mentioned in the standard except exponential and logarithmic functions. Finding the solutions approximately is limited to cases where $f(x)$ and $g(x)$ are polynomial functions.

[10] Tasks have a real-world context. In Algebra I, tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers. The focus in this module is on linear and exponential functions.

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Analyze functions using different representations.

F-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

F-IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

F-IF.C.9[12] Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Build new functions from existing functions.

F-BF.B.3[13] Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

Foundational Standards

Know that there are numbers that are not rational, and approximate them by rational numbers.

8.NS.A.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

[12] In Algebra I, tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers.

[13] In Algebra I, identifying the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative) is limited to linear and quadratic functions. Experimenting with cases and illustrating an explanation of the effects on the graph using technology is limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers. Tasks do not involve recognizing even and odd functions.

Work with radicals and integer exponents.

8.EE.A.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

Reason quantitatively and use units to solve problems.

N-Q.A.2[14] Define appropriate quantities for the purpose of descriptive modeling.★

N-Q.A.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.★

Create equations that describe numbers or relationships.

A-CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law* $V = IR$ *to highlight resistance* R.★

Understand solving equations as a process of reasoning and explain the reasoning.

A-REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Solve equations and inequalities in one variable.

A-REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Represent and solve equations and inequalities graphically.

A-REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Understand the concept of a function and use function notation.

F-IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

F-IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

[14] This standard will be assessed in Algebra I by ensuring that some modeling tasks (involving Algebra I content or securely held content from Grades 6–8) require the student to create a quantity of interest in the situation being described.

Build a function that models a relationship between two quantities.

F-BF.A.1[15] Write a function that describes a relationship between two quantities.★

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

Focus Standards for Mathematical Practice

MP.1 **Make sense of problems and persevere in solving them.** Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. In Module 4, students make sense of problems by analyzing the critical components of the problem, a verbal description, data set, or graph and persevere in writing the appropriate function to describe the relationship between two quantities.

MP.2 **Reason abstractly and quantitatively.** Mathematically proficient students make sense of quantities and their relationships in problem situations. This module alternates between algebraic manipulation of expressions and equations and interpretation of the quantities in the relationship in terms of the context. Students must be able to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own without necessarily attending to their referents, and then to *contextualize*—to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning requires the habit of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities (not just how to compute them), knowing different properties of operations, and using them with flexibility.

MP.4 **Model with mathematics.** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In this module, students create a function from a contextual situation described verbally, create a graph of their function, interpret key features of both the function and the graph (in the terms of the context), and answer questions related to the function and its graph. They also create a function from a data set based on a contextual situation. In Topic C, students use the full modeling cycle. They model quadratic functions presented mathematically or in a context. They explain the reasoning used in their writing or by using appropriate tools, such as graphing paper, graphing calculator, or computer software.

[15] Tasks have a real-world context. In Algebra I, tasks are limited to linear functions, quadratic functions, and exponential functions with domains in the integers.

MP.5 **Use appropriate tools strategically.** Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. Throughout the entire module, students must decide whether to use a tool to help find a solution. They must graph functions that are sometimes difficult to sketch (e.g., cube root and square root functions) and functions that are sometimes required to perform procedures that, when performed without technology, can be tedious and distract students from thinking mathematically (e.g., completing the square with non-integer coefficients). In such cases, students must decide when to use a tool to help with the calculation or graph so they can better analyze the model.

MP.6 **Attend to precision.** Mathematically proficient students try to communicate precisely to others. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. When calculating and reporting quantities in all topics of Module 4, students must be precise in choosing appropriate units and use the appropriate level of precision based on the information as it is presented. When graphing, they must select an appropriate scale.

MP.7 **Look for and make use of structure.** Mathematically proficient students look closely to discern a pattern or structure. They can see algebraic expressions as single objects, or as a composition of several objects. In this Module, students use the structure of expressions to find ways to rewrite them in different but equivalent forms. For example, in the expression $x^2 + 9x + 14$, students must see the 14 as 2×7 and the 9 as $2 + 7$ to find the factors of the quadratic. In relating an equation to a graph, they can see $y = -3(x-1)^2 + 5$ as 5 added to a negative number times a square and realize that its value cannot be more than 5 for any real domain value.

Terminology

New or Recently Introduced Terms

- **Axis of symmetry of the graph of a quadratic function** (Given a quadratic function in standard form, $f(x) = ax^2 + bx + c$, the vertical line given by the graph of the equation, $x = -\frac{b}{2a}$, is called the *axis of symmetry* of the graph of the quadratic function.)
- **Cube root function** (The parent function $f(x) = \sqrt[3]{x}$.)
- **Cubic function** (A polynomial function of degree 3.)
- **Degree of a monomial term** (The *degree of a monomial term* is the sum of the exponents of the variables that appear in a term of a polynomial.)
- **Degree of a polynomial** (The *degree of a polynomial in one variable in standard form* is the highest degree of the terms in the polynomial.)

- **Discriminant** (The *discriminant* of a quadratic function in the form $ax^2 + bx + c = 0$ is $b^2 - 4ac$. The nature of the roots of a quadratic equation can be identified by determining if the discriminant is positive, negative, or equal to zero.)
- **End behavior of a quadratic function** (Given a quadratic function in the form $f(x) = ax^2 + bx + c$ (or $f(x) = a(x-h)^2 + k$), the quadratic function is said to *open up* if $a > 0$ and *open down* if $a < 0$.)
- **Factored form for a quadratic function** (A quadratic function written in the form $f(x) = a(x-n)(x-m)$.)
- **Leading coefficient** (The *leading coefficient* of a polynomial is the coefficient of the term of highest degree.)
- **Parent function** (A *parent function* is the simplest function in a "family" of functions that can each be formed by one or more transformations of another.)
- **Quadratic formula** (The *quadratic formula* is the formula that emerges from solving the general form of a quadratic equation by completing the square, $y = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$. It can be used to solve any quadratic equation.)
- **Quadratic function** (A polynomial function of degree 2.)
- **Roots of a polynomial function** (The domain values for a polynomial function that make the value of the polynomial function equal zero when substituted for the variable.)

$$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0,$$

  where n is a non-negative integer, and $a_0, a_1, a_2, \ldots, a_n$ are constant coefficients with $a_n \neq 0$.)
- **Square root function** (The parent function $f(x) = \sqrt{x}$.)
- **Standard form for a quadratic function** (A quadratic function written in the form $f(x) = ax^2 + bx + c$.)
- **Standard form of a polynomial in one variable** (A polynomial expression with one variable symbol x is in *standard form* if it is expressed as,
- **Vertex form** (Completed-square form for a quadratic function; in other words, written in the form $f(x) = a(x-h)^2 + k$.)
- **Vertex of the graph of a quadratic function** (The point where the graph of a quadratic function and its axis of symmetry intersect is called the *vertex*. The vertex is either a maximum or a minimum of the quadratic function, depending on whether the leading coefficient of the function in standard form is negative or positive, respectfully.)

Familiar Terms and Symbols[16]

- Average rate of change
- Binomial
- Closed
- Closure
- Coefficient
- Cubic
- Cube root
- Degree of a polynomial
- Domain and range
- Explicit expression
- Factor
- Integers
- Irrational numbers
- Monomial
- Parabola
- Power
- Quadratic
- Rational numbers
- Real numbers
- Recursive process
- Solutions (solution set) of an equation
- Solution set
- Square root
- Term
- Trinomial
- Zeros of a function

Suggested Tools and Representations

- Coordinate Plane
- Equations
- Graphing calculator
- Graph paper

[16] These are terms and symbols students have seen previously.

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic A	Constructed response with rubric	A-SSE.A.1, A-SSE.A.2, A-SSE.B.3a, A-APR.A.1, A-CED.A.1, A-CED.A.2, A-REI.B.4b, A-REI.D.11, F-IF.B.4, F-IF.B.5, F-IF.B.6, F-IF.C.7a
End-of-Module Assessment Task	After Topic C	Constructed response with rubric	N-RN.B.3, A-SSE.A.1, A-SSE.A.2, A-SSE.B.3a, A-SSE.B.3b, A-APR.B.3, A-CED.A.1, A-CED.A.2, A-REI.B.4, F-IF.B.4, F-IF.B.6, F-IF.C.7a, F-IF.C.7b, F-IF.C.8a, F-IF.C.9, F-BF.B.3