Table of Contents[1]

Rational Numbers

[1] Each lesson is ONE day, and ONE day is considered a 45-minute period.

Grade 6 • Module 3

Rational Numbers

OVERVIEW

Students are familiar with the number line and determining the location of positive fractions, decimals, and whole numbers from previous grades. Students extend the number line (both horizontally and vertically) in Module 3 to include the opposites of whole numbers. The number line serves as a model to relate integers and other rational numbers to statements of order in real-world contexts. In this module's final topic, the number line model is extended to two-dimensions, as students use the coordinate plane to model and solve real-world problems involving rational numbers.

Topic A focuses on the development of the number line in the opposite direction (to the left or below zero). Students use positive integers to locate negative integers, understanding that a number and its opposite will be on opposite sides of zero and that both lie the same distance from zero. Students represent the opposite of a positive number as a negative number and vice-versa. Students realize that zero is its own opposite and that the opposite of the opposite of a number is actually the number itself (**6.NS.C.6a**). They use positive and negative numbers to represent real-world quantities, such as -50 to represent a $\$50$ debt or 50 to represent a $\$50$ deposit into a savings account (**6.NS.C.5**). Topic A concludes with students furthering their understanding of signed numbers to include the rational numbers. Students recognize that finding the opposite of any rational number is the same as finding an integer's opposite (**6.NS.C.6c**) and that two rational numbers that lie on the same side of zero will have the same sign, while those that lie on opposites sides of zero will have opposite signs.

In Topic B, students apply their understanding of a rational number's position on the number line (**6.NS.C.6c**) to order rational numbers. Students understand that when using a conventional horizontal number line, the numbers increase as you move along the line to the right and decrease as you move to the left. They recognize that if a and b are rational numbers and $a < b$, then it must be true that $-a > -b$. Students compare rational numbers using inequality symbols and words to state the relationship between two or more rational numbers. They describe the relationship between rational numbers in real-world situations and with respect to numbers' positions on the number line (**6.NS.C.7a**, **6.NS.C.7b**). For instance, students explain that $-10°\text{F}$ is warmer than $-11°\text{F}$ because -10 is to the right (or above) -11 on a number line and write $-10°\text{F} > -11°\text{F}$. Students use the concept of absolute value and its notation to show a number's distance from zero on the number line and recognize that opposite numbers have the same absolute value (**6.NS.C.7c**). In a real-world scenario, students interpret absolute value as magnitude for a positive or negative quantity. They apply their understanding of order and absolute value to determine that, for instance, a checking account balance that is less than -25 dollars represents a debt of more than $\$25$ (**6.NS.C.7d**).

In Topic C, students extend their understanding of the ordering of rational numbers in one dimension (on a number line) to the two-dimensional space of the coordinate plane. They construct the plane's vertical and horizontal axes, discovering the relationship between the four quadrants and the signs of the coordinates of points that lie in each quadrant (**6.NS.C.6b**, **6.NS.C.6c**). Students build upon their foundational understanding

from Grade 5 (**5.G.A.1**, **5.G.A.2**) of plotting points in the first quadrant and transition to locating points in all four quadrants. Students apply the concept of absolute value to find the distance between points located on vertical or horizontal lines and solve real-world problems related to distance, segments, and shapes (**6.NS.C.8**).

The 25-day module consists of 19 lessons; 6 days are reserved for administering the Mid- and End-of-Module Assessments, returning assessments, and remediating or providing further applications of the concepts. The Mid-Module Assessment follows Topic B, and the End-of-Module Assessment follows Topic C.

Focus Standards

Apply and extend previous understandings of numbers to the system of rational numbers.

6.NS.C.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

6.NS.C.6 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite.

b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.

c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

6.NS.C.7 Understand ordering and absolute value of rational numbers.

a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret* $-3 > -7$ *as a statement that* -3 *is located to the right of* -7 *on a number line oriented from left to right.*

b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write* $-3°\text{C} > -7°\text{C}$ *to express the fact that* $-3°\text{C}$ *is warmer than* $-7°\text{C}$.

c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of* -30 *dollars, write* $|-30| = 30$ *to describe the size of the debt in dollars.*

d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.*

6.NS.C.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Foundational Standards

Develop understanding of fractions as numbers.

3.NF.A.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.

a. Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.

b. Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

4.G.A.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Graph points on the coordinate plane to solve real-world and mathematical problems.

5.G.A.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).

5.G.A.2 Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Focus Standards for Mathematical Practice

MP.2 **Reason abstractly and quantitatively.** Students read a word problem involving integers, draw a number line or coordinate plane model, and write about their conclusions. They understand the meaning of quantities as they relate to the real world. For instance, a loss of 14 yards in a football game can be represented by -14, and a distance of 25 feet below sea level is greater than a distance of 5 feet above sea level because $|-25| > |5|$. Students decontextualize word problems related to distance by creating number lines and coordinate plane models. In doing so, they count the number of units between endpoints and use the concept of absolute value to justify their answer. For instance, when given the coordinate $(2, 6)$, students determine that the point $(2, -6)$ would be the same distance from the x-axis but in the opposite direction because both points have the same x-coordinate and their y-coordinates (6 and -6) have the same absolute value.

MP.4 **Model with mathematics.** Students use vertical and horizontal number lines to visualize integers and better understand their connection to whole numbers. They divide number line intervals into sub-intervals of tenths to determine the correct placement of rational numbers. Students may represent a decimal as a fraction or a fraction as a decimal to better understand its relationship to other rational numbers to which it is being compared. To explain the meaning of a quantity in a real-life situation (involving elevation, temperature, or direction), students may draw a diagram and/or number line to illustrate the location of the quantity in relation to zero or an established level that represents zero in that situation.

MP.6 **Attend to precision.** In representing signed numbers on a number line or as a quantity, students pay close attention to the direction and sign of a number. They realize that a negative number must lie to the left of zero on a horizontal number line or below zero on a vertical number line. They recognize that the way they represent their answer depends on the phrasing of a question and context of a word problem. For instance, a question that asks a student: "How many feet below sea level is the diver?" would require the answer to be stated as a positive number. Whereas, a question that is phrased: "Which integer would represent 40 feet below sea level?" would require the answer to be written as -40.

MP.7 **Look for and make use of structure.** Students understand the placement of negative numbers on a number line by observing the patterns that exist between negative and positive numbers with respect to zero. They recognize that two numbers are opposites if they are the same distance from zero and that zero is its own opposite. Students extend their understanding of the number line's structure to the coordinate plane to determine a point's location. They recognize the relationship between the signs of a point's coordinates and the quadrant in which the point lies.

Terminology

New or Recently Introduced Terms

- **Absolute Value** (The *absolute value* of a number is the distance between the number and zero on the number line. For example, $|3| = 3$, $|-4| = 4$, etc.)

- **Charge** (A *charge* is the amount of money a person must pay, as in a charge to an account, or a fee charged.)
- **Credit** (A *credit* is a decrease in an expense, as in money *credited* to an account. For instance, when a deposit is made into a checking account, the money is *credited* to the account. A credit is the opposite of a debit.)
- **Debit** (A *debit* is an increase in an expense or money paid out of an account. For instance, using a debit card to make a purchase will result in an expense, and money will be deducted from the related bank account.)
- **Deposit** (A *deposit* is the act of putting money into a bank account.)
- **Elevation** (*Elevation* is the height of a person, place, or thing above a certain reference level.)
- **Integers** (The numbers … $,-3, -2, -1, 0, 1, 2, 3,$ … are *integers* on the number line.)
- **Magnitude** (The *magnitude* is the absolute value of a measurement, given the measurement of a positive or negative quantity.)
- **Negative Number** (A *negative number* is a number less than zero.)
- **Opposite** (In a position on the other side; for example, negative numbers are the *opposite* direction from zero as positive numbers.)
- **Positive Number** (A *positive number* is a number greater than zero.)
- **Quadrants** (The four sections of the coordinate plane formed by the intersection of the axes are called *quadrants*.)
- **Rational Number** (A *rational number* is a fraction or the opposite of a fraction on the number line.)
- **Withdraw** (To *withdraw* is to take away; for example, to take money out of a bank account.)
- **Withdrawal** (A *withdrawal* is the act of taking money out of a bank account.)

Familiar Terms and Symbols[2]

- Coordinate Pair
- Coordinate Plane
- Fraction
- Line of Symmetry
- Ordered Pair
- Origin
- Quadrant
- Symmetry
- Whole Numbers
- x-Axis
- x-Coordinate
- y-Axis
- y-Coordinate

[2] These are terms and symbols students have seen previously.

Suggested Tools and Representations

- Horizontal and Vertical Number Lines
- Coordinate Plane

Sprints

Sprints are designed to develop fluency. They should be fun, adrenaline-rich activities that intentionally build energy and excitement. A fast pace is essential. During Sprint administration, teachers assume the role of athletic coaches. A rousing routine fuels students' motivation to do their personal best. Student recognition of increasing success is critical, and so every improvement is acknowledged. (See the Sprint Delivery Script for the suggested means of acknowledging and celebrating student success.)

One Sprint has two parts with closely-related problems on each. Students complete the two parts of the Sprint in quick succession with the goal of improving on the second part, even if only by one more.

Sprints are not to be used for a grade. Thus, there is no need for students to write their names on the Sprints. The low-stakes nature of the exercise means that even students with allowances for extended time can participate. When a particular student finds the experience undesirable, it is recommended that the student be allowed to opt-out and take the Sprint home. In this case, it is ideal if the student has a regular opportunity to express the desire to opt-in.

With practice, the Sprint routine takes about 8 minutes.

Sprint Delivery Script

Gather the following: stopwatch, a copy of Sprint A for each student, a copy of Sprint B for each student, answers for Sprint A and Sprint B. The following delineates a script for delivery of a pair of Sprints.

This sprint covers: *topic.*

Do not look at the Sprint, keep it turned face down on your desk.

There are xx problems on the Sprint. You will have 60 seconds. Do as many as you can. I do not expect any of you to finish.

On your mark, get set, GO.

60 *seconds of silence.*

STOP. Circle the last problem you completed.

I will read the answers. You say "YES" if your answer matches. Mark the ones you have wrong. Don't try to correct them.

Energetically, rapid-fire call the answers ONLY.

Stop reading answers after there are no more students answering, "Yes."

Fantastic! Count the number you have correct, and write it on the top of the page. This is your personal goal for Sprint B.

Raise your hand if you have one or more correct. Two or more, three or more...

Let us all applaud our runner up, [insert name] with x correct. And let us applaud our winner, [insert name], with x correct.

You have a few minutes to finish up the page and get ready for the next Sprint.

Students are allowed to talk and ask for help; let this part last as long as most are working seriously.

Stop working. I will read the answers again so you can check your work. You say "YES" if your answer matches.

Energetically, rapid-fire call the answers ONLY.

Optionally, ask students to stand and lead them in an energy-expanding exercise that also keeps the brain going. Examples are jumping jacks or arm circles, etc. while counting by 15*'s starting at* 15*, going up to* 150 *and back down to* 0*. You can follow this first exercise with a cool down exercise of a similar nature, such as calf raises with counting by one-sixths* $\left(\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}, 1 \ldots\right)$.

Hand out the second Sprint and continue reading the script.

Keep the Sprint face down on your desk.

There are xx problems on the Sprint. You will have 60 seconds. Do as many as you can. I do not expect any of you to finish.

On your mark, get set, GO.

60 *seconds of silence.*

STOP. Circle the last problem you completed.

I will read the answers. You say "YES" if your answer matches. Mark the ones you have wrong. Don't try to correct them.

Quickly read the answers ONLY.

Count the number you have correct, and write it on the top of the page.

Raise your hand if you have one or more correct. Two or more, three or more...

Let us all applaud our runner up, [insert name] with x correct. And let us applaud our winner, {insert name], with x correct.

Write the amount by which your score improved at the top of the page.

Raise your hand if you have one or more correct. Two or more, three or more...

Let us all applaud our runner up for most improved, [insert name]. And let us applaud our winner for most improved, [insert name].

You can take the Sprint home and finish it if you want.

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	6.NS.C.5, 6.NS.C.6a, 6.NS.C.6c, 6.NS.C.7
End-of-Module Assessment Task	After Topic C	Constructed response with rubric	6.NS.C.5, 6.NS.C.6a, 6.NS.C.6c, 6.NS.C.7, 6.NS.C.8