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Ratios and Proportional Relationships

[1] Each lesson is ONE day, and ONE day is considered a 45 minute period.

Grade 7 • Module 1

Ratios and Proportional Relationships

OVERVIEW

In Module 1, students build upon their Grade 6 reasoning about ratios, rates, and unit rates (**6.RP.A.1**, **6.RP.A.2**, **6.RP.3**) to formally define proportional relationships and the constant of proportionality (**7.RP.A.2**). In Topic A, students examine situations carefully to determine if they are describing a proportional relationship. Their analysis is applied to relationships given in tables, graphs, and verbal descriptions (**7.RP.A.2a**).

In Topic B, students learn that the unit rate of a collection of equivalent ratios is called the *constant of proportionality* and can be used to represent proportional relationships with equations of the form $y = kx$, where k is the constant of proportionality (**7.RP.A.2b**, **7.RP.A.2c**, **7.EE.B.4a**). Students relate the equation of a proportional relationship to ratio tables and to graphs and interpret the points on the graph within the context of the situation (**7.RP.A.2d**).

In Topic C, students extend their reasoning about ratios and proportional relationships to compute unit rates for ratios and rates specified by rational numbers, such as a speed of $\frac{1}{2}$ mile per $\frac{1}{4}$ hour (**7.RP.A.1**). Students apply their experience in the first two topics and their new understanding of unit rates for ratios and rates involving fractions to solve multistep ratio word problems (**7.RP.A.3**, **7.EE.B.4a**).

In the final topic of this module, students bring the sum of their experience with proportional relationships to the context of scale drawings (**7.RP.A.2b**, **7.G.A.1**). Given a scale drawing, students rely on their background in working with side lengths and areas of polygons (**6.G.A.1**, **6.G.A.3**) as they identify the scale factor as the constant of proportionality, calculate the actual lengths and areas of objects in the drawing, and create their own scale drawings of a two-dimensional view of a room or building. The topic culminates with a two-day experience of students creating a new scale drawing by changing the scale of an existing drawing.

Later in the year, in Module 4, students will extend the concepts of this module to percent problems.

The module is comprised of 22 lessons; 8 days are reserved for administering the Mid- and End-of-Module Assessments, returning the assessments, and remediating or providing further applications of the concepts. The Mid-Module Assessment follows Topic B. The End-of-Module Assessment follows Topic D.

Focus Standards

Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction ½ / ¼ miles per hour, equivalently 2 miles per hour.*

7.RP.A.2 Recognize and represent proportional relationships between quantities.

a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

c. Represent proportional relationships by equations. *For example, if total cost, t, is proportional to the number, n, of items purchased at a constant price, p, the relationship between the total cost and the number of items can be expressed as t = pn.*

d. Explain what a point *(x,y)* on the graph of a proportional relationship means in terms of the situation, with special attention to the points *(0,0)* and *(1,r)*, where *r* is the unit rate.

7.RP.A.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.B.4[2] Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

Draw, construct, and describe geometrical figures and describe the relationships between them.

7.G.A.1 Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

[2] In this module, the equations are derived from ratio problems. 7.EE.B.4a is returned to in Modules 2 and 3.

Foundational Standards

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.A.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*

6.RP.A.2 Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with *b ≠ 0*, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[3]

6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*

c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Solve real-world and mathematical problems involving area, surface area, and volume.

6.G.A.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

6.G.A.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

[3] Expectations for unit rates in this grade are limited to non-complex fractions.

Focus Standards for Mathematical Practice

MP.1 **Make sense of problems and persevere in solving them.** Students make sense of and solve multistep ratio problems, including cases with pairs of rational number entries; they use representations, such as ratio tables, the coordinate plane, and equations, and relate these representations to each other and to the context of the problem. Students depict the meaning of constant proportionality in proportional relationships, the importance of $(0,0)$ and $(1,r)$ on graphs and the implications of how scale factors magnify or shrink actual lengths of figures on a scale drawing.

MP.2 **Reason abstractly and quantitatively.** Students compute unit rates for paired data given in tables to determine if the data represents a proportional relationship. Use of concrete numbers will be analyzed to create and implement equations, including $y = kx$, where $k$is the constant of proportionality. Students decontextualize a given constant speed situation, representing symbolically the quantities involved with the formula, $distance = rate \times time$. In scale drawings, scale factors will be changed to create additional scale drawings of a given picture.

Terminology

New or Recently Introduced Terms

- **Proportional To** (Measures of one type of quantity are *proportional to* measures of a second type of quantity if there is a number $k > 0$ so that for every measure x of a quantity of the first type the corresponding measure y of a quantity of the second type is given by kx, i.e., $y = kx$.)
- **Proportional Relationship** (A one-to-one matching between two types of quantities such that the measures of quantities of the first type are proportional to the measures of quantities of the second type.)
- **Constant of Proportionality** (If a proportional relationship is described by the set of ordered pairs that satisfies the equation $y = kx$, where k is a positive constant, then k is called the *constant of proportionality*. For example, if the ratio of y to x is 2 to 3, then the constant of proportionality is $\frac{2}{3}$ and $y = \frac{2}{3}x$.)
- **One-to- One Correspondence** (Two figures in the plane, S and S', are said to be in one-to-one correspondence if there is a pairing between the points in S and S', so that each point P of S is paired with one and only one point P' in S', and likewise, each point Q' in S' is paired with one and only one point Q in S.)
- **Scale Drawing and Scale Factor**[4] (For two figures in the plane, S and S', S' is said to be a *scale drawing* of S with *scale factor* r if there exists a one-to-one correspondence between S and S' so that under the pairing of this one-to-one correspondence, the distance $|PQ|$ between any two points P and Q of S is related to the distance $|P'Q'|$ between corresponding points P' and Q' of S' by $|P'Q'| = r|PQ|$.)

[4] These terms will be formally defined in Grade 8. A description is provided in Grade 7.

Familiar Terms and Symbols[5]

- Ratio
- Rate
- Unit Rate
- Equivalent Ratio
- Ratio Table

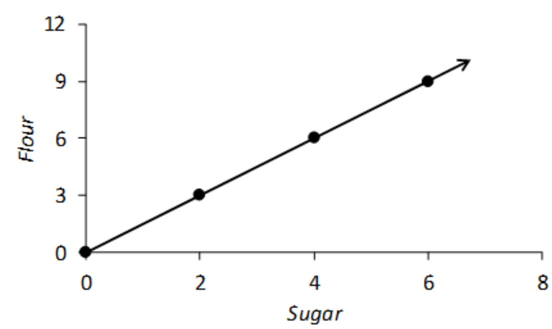
Suggested Tools and Representations

- Ratio Table (See example below)
- Coordinate Plane (See example below)
- Equations of the form $y = kx$

Ratio Table

Sugar	*Flour*
2	3
4	6
6	9

Coordinate Plane



Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	7.RP.A.2
End-of-Module Assessment Task	After Topic D	Constructed response with rubric	7.RP.A.1, 7.RP.A.2, 7.RP.A.3, 7.EE.B.4a, 7.G.A.1

[5] These are terms and symbols students have seen previously.