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Expressions and Equations

[1] Each lesson is ONE day, and ONE day is considered a 45-minute period.

Grade 7 • Module 3

Expressions and Equations

OVERVIEW

In Grade 6, students interpreted expressions and equations as they reasoned about one-variable equations (**6.EE.A.2**). This module consolidates and expands upon students' understanding of equivalent expressions as they apply the properties of operations (associative, commutative, and distributive) to write expressions in both standard form (by expanding products into sums) and in factored form (by expanding sums into products). They use linear equations to solve unknown angle problems and other problems presented within context to understand that solving algebraic equations is all about the numbers. It is assumed that a number already exists to satisfy the equation and context; we just need to discover it. A number sentence is an equation that is said to be true if both numerical expressions evaluate to the same number; it is said to be false otherwise. Students use the number line to understand the properties of inequality and recognize when to *preserve the inequality* and when to *reverse the inequality* when solving problems leading to inequalities. They interpret solutions within the context of problems. Students extend their sixth-grade study of geometric figures and the relationships between them as they apply their work with expressions and equations to solve problems involving area of a circle and composite area in the plane, as well as volume and surface area of right prisms. In this module, students discover the most famous ratio of all, π, and begin to appreciate why it has been chosen as the symbol to represent the Grades 6–8 mathematics curriculum, *A Story of Ratios*.

To begin this module, students will generate equivalent expressions using the fact that addition and multiplication can be done in any order with any grouping and will extend this understanding to subtraction (adding the inverse) and division (multiplying by the multiplicative inverse, also known as the reciprocal) (**7.EE.A.1**). They extend the properties of operations with numbers (learned in earlier grades) and recognize how the same properties hold true for letters that represent numbers. Knowledge of rational number operations from Module 2 is demonstrated as students collect like terms containing both positive and negative integers.

An area model is used as a tool for students to rewrite products as sums and sums as products and to provide a visual representation leading students to recognize the repeated use of the distributive property in factoring and expanding linear expressions (**7.EE.A.1**). Students examine situations where more than one form of an expression may be used to represent the same context, and they see how looking at each form can bring a new perspective (and thus deeper understanding) to the problem. Students recognize and use the identity properties and the existence of additive inverses to efficiently write equivalent expressions in standard form, for example, $2x + (-2x) + 3 = 0 + 3 = 3$ (**7.EE.A.2**). By the end of the topic, students have the opportunity to practice knowledge of operations with rational numbers gained in Module 2 (**7.NS.A.1**, **7.NS.A.2**) as they collect like terms with rational number coefficients (**7.EE.A.1**).

In Topic B, students use linear equations and inequalities to solve problems (**7.EE.B.4**). They continue to use tape diagrams from earlier grades where they see fit, but will quickly discover that some problems would more reasonably be solved algebraically (as in the case of large numbers). Guiding students to arrive at this realization on their own develops the need for algebra. This algebraic approach builds upon work in Grade 6 with equations (**6.EE.B.6**, **6.EE.B.7**) to now include multi-step equations and inequalities containing rational numbers (**7.EE.B.3**, **7.EE.B.4**). Students solve problems involving consecutive numbers; total cost; age comparisons; distance, rate, and time; area and perimeter; and missing angle measures. Solving equations with a variable is all about numbers, and students are challenged with the goal of finding the number that makes the equation true. When given in context, students recognize that a value exists, and it is simply their job to discover what that value is. Even the angles in each diagram have a precise value, which can be checked with a protractor to ensure students that the value they find does indeed create a true number sentence.

In Topic C, students continue work with geometry as they use equations and expressions to study area, perimeter, surface area, and volume. This final topic begins by modeling a circle with a bicycle tire and comparing its perimeter (one rotation of the tire) to the length across (measured with a string) to allow students to discover the most famous ratio of all, pi. Activities in comparing circumference to diameter are staged precisely for students to recognize that this symbol has a distinct value and can be approximated by $\frac{22}{7}$, or 3.14, to give students an intuitive sense of the relationship that exists. In addition to representing this value with the π symbol, the fraction and decimal approximations allow for students to continue to practice their work with rational number operations. All problems are crafted in such a way as to allow students to practice skills in reducing within a problem, such as using $\frac{22}{7}$ for finding circumference with a given diameter length of 14 cm, and recognize what value would be best to approximate a solution. This understanding allows students to accurately assess work for reasonableness of answers. After discovering and understanding the value of this special ratio, students will continue to use pi as they solve problems of area and circumference (**7.G.B.4**).

In this topic, students derive the formula for area of a circle by dividing a circle of radius r into pieces of pi and rearranging the pieces so that they are lined up, alternating direction, and form a shape that resembles a rectangle. This "rectangle" has a length that is $\frac{1}{2}$ the circumference and a width of r. Students determine that the area of this rectangle (reconfigured from a circle of the same area) is the product of its length and its width: $\frac{1}{2}C \cdot r = \frac{1}{2}2\pi r \cdot r = \pi r^2$ (**7.G.B.4**). The precise definitions for diameter, circumference, pi, and circular region or disk will be developed during this topic with significant time being devoted to students' understanding of each term.

Students build upon their work in Grade 6 with surface area and nets to understand that surface area is simply the sum of the area of the lateral faces and the base(s) (**6.G.A.4**). In Grade 7, they continue to solve real-life and mathematical problems involving area of two-dimensional shapes and surface area and volume of prisms, e.g., rectangular, triangular, focusing on problems that involve fractional values for length (**7.G.B.6**). Additional work (examples) with surface area will occur in Module 6 after a formal definition of rectangular pyramid is established.

This module is comprised of 26 lessons; 9 days are reserved for administering the Mid-Module and End-of-Module Assessments, returning the assessments, and remediating or providing further applications of the concepts. The Mid-Module Assessment follows Topic B, and the End-of-Module Assessment follows Topic C.

Focus Standards

Use properties of operations to generate equivalent expressions.

7.EE.A.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

7.EE.A.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example,* $a + 0.05a = 1.05a$ *means that "increase by* 5%*" is the same as "multiply by* 1.05*."*

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.B.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making* $\$25$ *an hour gets a* 10% *raise, she will make an additional* $1/10$ *of her salary an hour, or* $\$2.50$*, for a new salary of* $\$27.50$*. If you want to place a towel bar* $9\ 3/4$ *inches long in the center of a door that is* $27\ 1/2$ *inches wide, you will need to place the bar about* 9 *inches from each edge; this estimate can be used as a check on the exact computation*.

7.EE.B.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is* 54 *cm. Its length is* 6 *cm. What is its width?*

b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid* $\$50$ *per week plus* $\$3$ *per sale. This week you want your pay to be at least* $\$100$*. Write an inequality for the number of sales you need to make, and describe the solutions.*

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.G.B.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

7.G.B.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

7.G.B.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Foundational Standards

Understand and apply properties of operations and the relationship between addition and subtraction

1.OA.B.3 Apply properties of operations as strategies to add and subtract.[2] *Examples: If* $8 + 3 = 11$ *is known, then* $3 + 8 = 11$ *is also known. (Commutative property of addition.) To add* $2 + 6 + 4$*, the second two numbers can be added to make a ten, so* $2 + 6 + 4 = 2 + 10 = 12$*. (Associative property of addition.)*

Understand properties of multiplication and the relationship between multiplication and division.

3.OA.B.5 Apply properties of operations as strategies to multiply and divide.[2] *Examples: If* $6 \times 4 = 24$ *is known, then* $4 \times 6 = 24$ *is also known. (Commutative property of multiplication.)*
$3 \times 5 \times 2$ *can be found by* $3 \times 5 = 15$*, then* $15 \times 2 = 30$*, or by* $5 \times 2 = 10$*, then* $3 \times 10 = 30$*. (Associative property of multiplication.) Knowing that* $8 \times 5 = 40$ *and* $8 \times 2 = 16$*, one can find* 8×7 *as* $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$*. (Distributive property.)*

Geometric measurement: understand concepts of angle and measure angles.

4.MD.C.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $1/360$ of a circle is called a "one-degree angle," and can be used to measure angles.

b. An angle that turns through n one-degree angles is said to have an angle measure of n degrees.

4.MD.C.6 Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

[2] Students need not use formal terms for these properties.

4.MD.C.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

Apply and extend previous understandings of arithmetic to algebraic expressions.

6.EE.A.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression* $3(2+x)$ *to produce the equivalent expression* $6+3x$*; apply the distributive property to the expression* $24x+18y$ *to produce the equivalent expression* $6(4x+3y)$*; apply properties of operations to* $y+y+y$ *to produce the equivalent expression* $3y$*.*

6.EE.A.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions* $y+y+y$ *and* $3y$ *are equivalent because they name the same number regardless of which number* y *stands for.*

Reason about and solve one-variable equations and inequalities.

6.EE.B.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

6.EE.B.7 Solve real-world and mathematical problems by writing and solving equations in the form $x+p=q$ and $px=q$ for cases in which p, q, and x are all nonnegative rational numbers.

6.EE.B.8 Write an inequality of the form $x>c$ or $x<c$ to represent a constraint or condition in a real-world mathematical problem. Recognize that inequalities of the form $x>c$ or $x<c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Solve real-world and mathematical problems involving area, surface area, and volume.

6.G.A.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

6.G.A.2 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V=lwh$ and $V=bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

6.G.A.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

7.NS.A.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has* 0 *charge because its two constituents are oppositely charged.*

b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

d. Apply properties of operations as strategies to add and subtract rational numbers.

7.NS.A.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.

c. Apply properties of operations as strategies to multiply and divide rational numbers.

d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

Focus Standards for Mathematical Practice

MP.2 **Reason abstractly and quantitatively**. Students make sense of how quantities are related within a given context and formulate algebraic equations to represent this relationship. They use the properties of operations to manipulate the symbols that are used in place of numbers, in particular, pi. In doing so, students reflect upon each step in solving and recognize that these properties hold true since the variable is really just holding the place for a number. Students analyze solutions and connect back to ensure reasonableness within context.

MP.4 **Model with mathematics**. Throughout the module, students use equations and inequalities as models to solve mathematical and real-world problems. In discovering the relationship between circumference and diameter in a circle, they will use real objects to analyze the relationship and draw conclusions. Students test conclusions with a variety of objects to see if the results hold true, possibly improving the model if it has not served its purpose.

MP.6 **Attend to precision**. Students are precise in defining variables. They understand that a variable represents one number. They use appropriate vocabulary and terminology when communicating about expressions, equations, and inequalities. They use the definition of equation from Grade 6 to understand how to use the equal sign consistently and appropriately. Circles and related notions about circles are precisely defined in this module.

MP.7 **Look for and make use of structure.** Students recognize the repeated use of the distributive property as they write equivalent expressions. Students recognize how equations leading to the form $px + q = r$ and $p(x + q) = r$ are useful in solving a variety of problems. They see patterns in the way that these equations are solved. Students apply this structure as they understand the similarities and differences in how an inequality of the type $px + q > r$ or $px + q < r$ is solved.

MP.8 **Look for and express regularity in repeated reasoning**. Students use area models to write products as sums and sums as products and recognize how this model is a way to organize results from repeated use of the distributive property. As students work to solve problems, they maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of solutions as they are represented in contexts that allow for students to know that they found the intended value for a given variable. As they solve problems involving pi, they notice how a problem may be reduced by using a given estimate for pi to make calculations more efficient.

Terminology

New or Recently Introduced Terms

- **An Expression in Expanded Form** (description) (An expression that is written as sums (and/or differences) of products whose factors are numbers, variables, or variables raised to whole number powers is said to be in *expanded form*. A single number, variable, or a single product of numbers and/or variables is also considered to be in expanded form.)
- **An Expression in Factored Form** (middle school description) (An expression that is a product of two or more expressions is said to be in *factored form*.)
- **An Expression in Standard Form** (description) (An expression that is in expanded form where all like terms have been collected is said to be in *standard form.)*
- **Coefficient of the Term** (The number found by multiplying just the numbers in a term together is called the *coefficient of the term*.)
- **Circle** (Given a point C in the plane and a number $r > 0$, the *circle* with center C and radius r is the set of all points in the plane that are distance r from the point C.)

- **Diameter of a Circle** (*The diameter of a circle* is the length of any segment that passes through the center of a circle whose endpoints lie on the circle. If r is the radius of a circle, then the diameter is $2r$.)
- **Circumference** (The *circumference* is the length around a circle.)[3]
- **Pi** (The number *pi*, denoted π, is the value of the ratio given by the circumference to the diameter in a circle; that is, $\pi =$ (circumference)/(diameter).)
- **Circular Region or Disk** (Given a point C in the plane and a number $r > 0$, the *circular region* (or disk) with center C and radius r is the set of all points in the plane whose distance from the point C is less than or equal to r. The interior of a circle with center C and radius r is the set of all points in the plane whose distance from the point C is less than r.)

Familiar Terms and Symbols[4]

- Adjacent Angles
- Cube
- Distribute
- Equation
- Equivalent Expressions
- Expression (middle school description)
- Factor
- Figure
- Identity
- Inequality
- Length of a Segment
- Linear Expression
- Measure of an Angle
- Number Sentence
- Numerical Expression (middle school description)
- Properties of Operations (distributive, commutative, associative)
- Right Rectangular Prism
- Segment
- Square
- Surface of a Prism
- Term
- Triangle
- True or False Number Sentence

[3] "Distance around a circular arc" is taken as an undefined term in G-CO.A.1.
[4] These are terms and symbols students have seen previously.

- Truth Values of a Number Sentence
- Value of a Numerical Expression
- Variable (middle school description)
- Vertical Angles

Suggested Tools and Representations

- Area Model
- Coordinate Plane
- Equations and Inequalities
- Expressions
- Geometric Figures
- Nets for Three-Dimensional Figures
- Number Line
- Protractor
- Tape Diagram

Sprints

Sprints are designed to develop fluency. They should be fun, adrenaline-rich activities that intentionally build energy and excitement. A fast pace is essential. During Sprint administration, teachers assume the role of athletic coaches. A rousing routine fuels students' motivation to do their personal best. Student recognition of increasing success is critical, and so every improvement is acknowledged. (See the Sprint Delivery Script for the suggested means of acknowledging and celebrating student success.)

One Sprint has two parts with closely-related problems on each. Students complete the two parts of the Sprint in quick succession with the goal of improving on the second part, even if only by one more.

Sprints are not to be used for a grade. Thus, there is no need for students to write their names on the Sprints. The low-stakes nature of the exercise means that even students with allowances for extended time can participate. When a particular student finds the experience undesirable, it is recommended that the student be allowed to opt-out and take the Sprint home. In this case, it is ideal if the student has a regular opportunity to express the desire to opt-in.

With practice, the Sprint routine takes about 8 minutes.

Sprint Delivery Script

Gather the following: stopwatch, a copy of Sprint A for each student, a copy of Sprint B for each student, answers for Sprint A and Sprint B. The following delineates a script for delivery of a pair of Sprints.

This sprint covers: *topic.*

Do not look at the Sprint, keep it turned face down on your desk.

There are xx problems on the Sprint. You will have 60 seconds. Do as many as you can. I do not expect any of you to finish.

On your mark, get set, GO.

60 seconds of silence.

STOP. Circle the last problem you completed.

I will read the answers. You say "YES" if your answer matches. Mark the ones you have wrong. Don't try to correct them.

Energetically, rapid-fire call the answers ONLY.

Stop reading answers after there are no more students answering, "Yes."

Fantastic! Count the number you have correct, and write it on the top of the page. This is your personal goal for Sprint B.

Raise your hand if you have 1 or more correct. 2 or more, 3 or more ...

Let us all applaud our runner up, [insert name] with x correct. And let us applaud our winner, [insert name], with x correct.

You have a few minutes to finish up the page and get ready for the next Sprint.

Students are allowed to talk and ask for help; let this part last as long as most are working seriously.

Stop working. I will read the answers again so you can check your work. You say "YES" if your answer matches.

Energetically, rapid-fire call the answers ONLY.

Optionally, ask students to stand and lead them in an energy-expanding exercise that also keeps the brain going. Examples are jumping jacks or arm circles, etc. while counting by 15's starting at 15, going up to 150 and back down to 0. You can follow this first exercise with a cool down exercise of a similar nature, such as calf raises with counting by one-sixths $\left(\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}, 1, \ldots\right)$.

Hand out the second Sprint and continue reading the script.

Keep the Sprint face down on your desk.

There are xx problems on the Sprint. You will have 60 seconds. Do as many as you can. I do not expect any of you to finish.

On your mark, get set, GO.

60 seconds of silence.

STOP. Circle the last problem you completed.

I will read the answers. You say "YES" if your answer matches. Mark the ones you have wrong. Don't try to correct them.

Quickly read the answers ONLY.

Count the number you have correct, and write it on the top of the page.

Raise your hand if you have 1 or more correct. 2 or more, 3 or more, ...

Let us all applaud our runner up, [insert name] with x correct. And let us applaud our winner, [insert name], with x correct.

Write the amount by which your score improved at the top of the page.

Raise your hand if you improved your score by 1 or more. 2 or more, 3 or more ...

Let us all applaud our runner up for most improved, [insert name]. And let us applaud our winner for most improved, [insert name].

You can take the Sprint home and finish it if you want.

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	7.EE.A.1, 7.EE.A.2, 7.EE.B.3, 7.EE.B.4, 7.G.B.5
End-of-Module Assessment Task	After Topic C	Constructed response with rubric	7.EE.A.1, 7.EE.A.2, 7.G.B.4, 7.G.B.5, 7.G.B.6