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¹ Each lesson is ONE day, and ONE day is considered a 45-minute period.

Grade 8 • Module 3

Similarity

OVERVIEW

In Module 3, students learn about dilation and similarity and apply that knowledge to a proof of the Pythagorean Theorem based on the Angle-Angle criterion for similar triangles. The module begins with the definition of dilation, properties of dilations and compositions of dilations. The instruction regarding dilation in Module 3 is structured similarly to the instruction regarding concepts of basic rigid motions in Module 2. One overarching goal of this module is to replace the common idea of “same shape, different sizes” with a definition of similarity that can be applied to geometric shapes that are not polygons, such as ellipses and circles.

In this module, students describe the effect of dilations on two-dimensional figures in general and using coordinates. Building on prior knowledge of scale drawings (**7.G.A.1**), Module 3 demonstrates the effect dilation has on a figure when the scale factor is greater than zero but less than one (shrinking of figure), equal to one (congruence) and greater than one (magnification of figure). Once students understand how dilation transforms figures in the plane, they examine the effect that dilation has on points and figures in the coordinate plane. Beginning with points, students learn the multiplicative effect that dilation has on the coordinates of an ordered pair. Then students apply the knowledge about points to describe the effect dilation has on figures in the coordinate plane, in terms of their coordinates.

Additionally, Module 3 demonstrates that a two-dimensional figure is similar to another if the second can be obtained from a dilation followed by congruence. Knowledge of basic rigid motions is reinforced throughout the module, specifically when students describe the sequence that exhibits a similarity between two given figures. In Module 2, students used vectors to describe the translation of the plane. Module 3 begins in the same way, but once figures are bound to the coordinate plane, students will describe translations in terms of units left or right and units up or down. When figures on the coordinate plane are rotated, the center of rotation is the origin of the graph. In most cases, students will describe the rotation as having center O and degree d , unless the rotation can be easily identified, i.e., a rotation of 90° or 180° . Reflections remain reflections across a line, but when possible, students should identify the line of reflection as the x -axis or y -axis.

It should be noted that congruence, together with similarity, is *the* fundamental concept in planar geometry. It is a concept defined without coordinates. In fact, it is most transparently understood when introduced without the extra conceptual baggage of a coordinate system. This is partly because a coordinate system picks out a preferred point (the origin), which then centers most discussions of rotations, reflections, and translations at or in reference to that point. These discussions are further restricted to only the “nice” rotations, reflections, or translations that are easy to do in a coordinate plane. Restricting to “nice” transformations is a huge mistake mathematically because it is antithetical to the main points that must be made about congruence: that rotations, translations, and reflections are abundant in the plane; that for every point in the plane, there are an *infinite number* of rotations up to 360° , that for every line in the plane

there is a reflection, and that for every directed line segment there is a translation. It is this abundance that helps students realize that every congruence transformation (i.e., the act of “picking up a figure” and moving it to another location) can be accomplished through a sequence of translations, rotations, and reflections and further, that similarity is a dilation followed by a congruence transformation.

In Grades 6 and 7, students learned about unit rate, rates in general (**6.RP.A.2**), and how to represent and use proportional relationships between quantities (**7.RP.A.2**, **7.RP.A.3**). In Module 3, students apply this knowledge of proportional relationships and rates to determine if two figures are similar, and if so, by what scale factor one can be obtained from the other. By looking at the effect of a scale factor on the length of a segment of a given figure, students will write proportions to find missing lengths of similar figures.

Module 3 provides another opportunity for students to learn about the Pythagorean Theorem and its applications in these extension lessons. With the concept of similarity firmly in place, students are shown a proof of the Pythagorean Theorem that uses similar triangles.

Focus Standards

Understand congruence and similarity using physical models, transparencies, or geometry software.

- 8.G.A.3** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
- 8.G.A.4** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
- 8.G.A.5** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

Understand and apply the Pythagorean Theorem.

- 8.G.B.6** Explain a proof of the Pythagorean Theorem and its converse.
- 8.G.B.7** Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Foundational Standards

Understand ratio concepts and use ratio reasoning to solve problems.

- 6.RP.A.2** Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $3/4$ cup of flour for each cup of sugar." "We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger."*²

Analyze proportional relationships and use them to solve real-world and mathematical problems.

- 7.RP.A.2** Recognize and represent proportional relationships between quantities.
- 7.RP.A.3** Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

Draw, construct, and describe geometrical figures and describe the relationships between them.

- 7.G.A.1** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
- 7.G.A.2** Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

Focus Standards for Mathematical Practice

- MP.3** **Construct viable arguments and critique the reasoning of others.** Many times in this module, students are exposed to the reasoned logic of proofs. Students are called on to make conjectures about the effect of dilations on angles, rays, lines, and segments, and then they must evaluate the validity of their claims based on evidence. Students also make conjectures about the effect of dilation on circles, ellipses, and other figures. Students are encouraged to participate in discussions and evaluate the claims of others.
- MP.4** **Model with mathematics.** This module provides an opportunity for students to apply their knowledge of dilation and similarity in real-world applications. Students will use shadow lengths and a known height to find the height of trees, the distance across a lake, and the height of a flagpole.
- MP.6** **Attend to precision.** To communicate precisely, students will use clear definitions in discussions with others and in their own reasoning with respect to similar figures. Students will use the basic properties of dilations to prove or disprove claims about a pair of figures.

² Expectations for unit rates in this grade are limited to non-complex fractions.

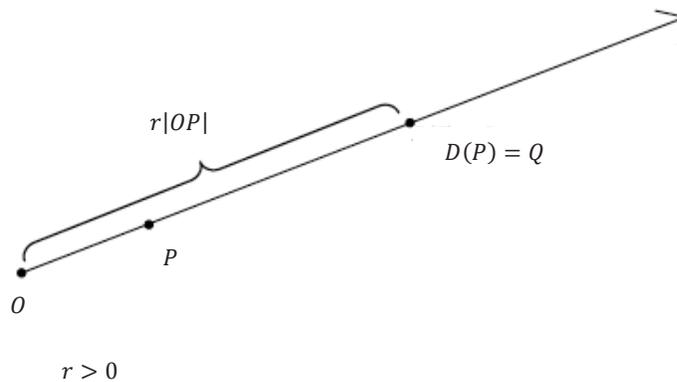
Students will incorporate their knowledge about basic rigid motions as it relates to similarity, specifically in the description of the sequence that is required to prove two figures are similar.

- MP.8** **Look for and express regularity in repeated reasoning.** Students will look at multiple examples of dilations with different scale factors. Then students explore dilations to determine what scale factor to apply to return a figure dilated by a scale factor r to its original size.

Terminology

New or Recently Introduced Terms

- Dilation** (*Dilation*, D , is a transformation of the plane with center O and scale factor r ($r > 0$). If $D(O) = O$ and if $P \neq O$, then the point $D(P)$, to be denoted by Q , is the point on the ray \overrightarrow{OP} so that $|OQ| = r|OP|$. If the scale factor $r \neq 1$, then a dilation in the coordinate plane is a transformation that shrinks or magnifies a figure by multiplying each coordinate of the figure by the scale factor.)



- Congruence** (A finite composition of basic rigid motions—reflections, rotations, translations—of the plane. Two figures in a plane are *congruent* if there is a congruence that maps one figure onto the other figure.)
- Similar** (Two figures in the plane are *similar* if there exists a similarity transformation taking one figure to the other.)
- Similarity Transformation** (A *similarity transformation*, or *similarity*, is a composition of a finite number of basic rigid motions or dilations. The scale factor of a similarity transformation is the product of the scale factors of the dilations in the composition; if there are no dilations in the composition, the scale factor is defined to be 1.)
- Similarity** (A *similarity* is an example of a transformation.)

Familiar Terms and Symbols³

- Scale Drawing
- Angle-Preserving

Suggested Tools and Representations

- Compass (required)
- Transparency or patty paper
- Wet or dry erase markers for use with transparency
- Geometry software (optional)
- Ruler
- Protractor
- Video that demonstrates Pythagorean Theorem proof using similar triangles:
<http://www.youtube.com/watch?v=QCyxYLFsfU>

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic A	Constructed response with rubric	8.G.A.3
End-of-Module Assessment Task	After Topic B	Constructed response with rubric	8.G.A.3, 8.G.A.4, 8.G.A.5

³ These are terms and symbols students have seen previously.