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Linear and Exponential Functions

[1] Each lesson is ONE day, and ONE day is considered a 45-minute period.

Algebra I • Module 3

Linear and Exponential Functions

OVERVIEW

In earlier grades, students define, evaluate, and compare functions and use them to model relationships between quantities (**8.F.A.1**, **8.F.A.2**, **8.F.A.3**, **8.F.B.4**, **8.F.B.5**). In this module, students extend their study of functions to include function notation and the concepts of domain and range. They explore many examples of functions and their graphs, focusing on the contrast between linear and exponential functions. They interpret functions given graphically, numerically, symbolically, and verbally; translate between representations; and understand the limitations of various representations.

In Topic A, students explore arithmetic and geometric sequences as an introduction to the formal notation of functions (**F-IF.A.1**, **F-IF.A.2**). They interpret arithmetic sequences as linear functions with integer domains and geometric sequences as exponential functions with integer domains (**F-IF.A.3**, **F-BF.A.1a**). Students compare and contrast the rates of change of linear and exponential functions, looking for structure in each and distinguishing between additive and multiplicative change (**F-IF.B.6**, **F-LE.A.1**, **F-LE.A.2**, **F-LE.A.3**).

In Topic B, students connect their understanding of functions to their knowledge of graphing from Grade 8. They learn the formal definition of a function and how to recognize, evaluate, and interpret functions in abstract and contextual situations (**F-IF.A.1**, **F-IF.A.2**). Students examine the graphs of a variety of functions and learn to interpret those graphs using precise terminology to describe such key features as domain and range, intercepts, intervals where the function is increasing or decreasing, and intervals where the function is positive or negative (**F-IF.A.1**, **F-IF.B.4**, **F-IF.B.5**, **F-IF.C.7a**).

In Topic C, students extend their understanding of piecewise functions and their graphs including the absolute value and step functions. They learn a graphical approach to circumventing complex algebraic solutions to equations in one variable, seeing them as $f(x) = g(x)$ and recognizing that the intersection of the graphs of $f(x)$ and $g(x)$ are solutions to the original equation (**A-REI.D.11**). Students use the absolute value function and other piecewise functions to investigate transformations of functions and draw formal conclusions about the effects of a transformation on the function's graph (**F-IF.C.7**, **F-BF.B.3**).

Finally, in Topic D, students apply and reinforce the concepts of the module as they examine and compare exponential, piecewise, and step functions in a real-world context (**F-IF.C.9**). They create equations and functions to model situations (**A-CED.A.1**, **F-BF.A.1**, **F-LE.A.2**), rewrite exponential expressions to reveal and relate elements of an expression to the context of the problem (**A-SSE.B.3c**, **F-LE.B.5**), and examine the key features of graphs of functions, relating those features to the context of the problem (**F-IF.B.4**, **F-IF.B.6**).

The Mid-Module Assessment follows Topic B. The End-of-Module Assessment follows Topic D.

Focus Standards

Write expressions in equivalent forms to solve problems.

A-SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★

c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $\left(1.15^{1/12}\right)^{12t} \approx 1.01212t$ *to reveal the approximate equivalent monthly interest rate if the annual rate is* 15%.[2]

Create equations that describe numbers or relationships.

A-CED.A.1[3] Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*.★

Represent and solve equations and inequalities graphically.

A-REI.D.11[4] Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★

Understand the concept of a function and use function notation.

F-IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

F-IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F-IF.A.3[5] Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.

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[2] Tasks have a real-world context. As described in the standard, there is an interplay between the mathematical structure of the expression and the structure of the situation such that choosing and producing an equivalent form of the expression reveals something about the situation. In Algebra I, tasks are limited to exponential expressions with integer exponents.
[3] In Algebra I, tasks are limited to linear, quadratic, or exponential equations with integer exponents.
[4] In Algebra I, tasks that assess conceptual understanding of the indicated concept may involve any of the function types mentioned in the standard except exponential and logarithmic functions. Finding the solutions approximately is limited to cases where *f*(*x*) and *g*(*x*) are polynomial functions.
[5] This standard is part of the Major Content in Algebra I and will be assessed accordingly.

Interpret functions that arise in applications in terms of the context.

F-IF.B.4[6] For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.★

F-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

F-IF.B.6[7] Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

Analyze functions using different representations.

F-IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

F-IF.C.9[8] Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Build a function that models a relationship between two quantities.

F-BF.A.1[9] Write a function that describes a relationship between two quantities.★

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

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[6] Tasks have a real-world context. In Algebra I, tasks are limited to linear functions, quadratic functions, square-root functions, cube-root functions, piecewise functions (including step functions and absolute-value functions), and exponential functions with domains in the integers.

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[9] Tasks have a real-world context. In Algebra I, tasks are limited to linear functions, quadratic functions, and exponential functions with domains in the integers.

Build new functions from existing functions.

F-BF.B.3[10] Identify the effect on the graph of replacing $f(x)$ by $f(x)+k$, $k\,f(x)$, $f(kx)$, and $f(x+k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

Construct and compare linear, quadratic, and exponential models and solve problems.

F-LE.A.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.★

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F-LE.A.2[11] Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).★

F-LE.A.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.★

Interpret expressions for functions in terms of the situation they model.

F-LE.B.5[12] Interpret the parameters in a linear or exponential function in terms of a context.★

Foundational Standards

Work with radicals and integer exponents.

8.EE.A.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

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[10] In Algebra I, identifying the effect on the graph of replacing $f(x)$ by $f(x)+k$, $k\,f(x)$, $f(kx)$, and $f(x+k)$ for specific values of k (both positive and negative) is limited to linear and quadratic functions. Experimenting with cases and illustrating an explanation of the effects on the graph using technology is limited to linear functions, quadratic functions, square-root functions, cube-root functions, piecewise functions (including step functions and absolute-value functions), and exponential functions with domains in the integers. Tasks do not involve recognizing even and odd functions. The focus in this module is on linear and exponential functions.

[11] In Algebra I, tasks are limited to constructing linear and exponential functions in simple (e.g., not multi-step) context.

[12] Tasks have a real-world context. In Algebra I, exponential functions are limited to those with domains in the integers.

8.EE.A.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

Define, evaluate, and compare functions.

8.F.A.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[13]

8.F.A.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

8.F.A.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points $(1,1)$, $(2,4)$ and $(3,9)$, which are not on a straight line.*

Use functions to model relationships between quantities.

8.F.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

8.F.B.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Reason quantitatively and use units to solve problems.

N-Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

N-Q.A.2[14] Define appropriate quantities for the purpose of descriptive modeling.

N-Q.A.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

[13] Function notation is not required in Grade 8.
[14] This standard will be assessed in Algebra I by ensuring that some modeling tasks (involving Algebra I content or securely held content from Grades 6–8) require the student to create a quantity of interest in the situation being described.

Interpret the structure of expressions.

A-SSE.A.1 Interpret expressions that represent a quantity in terms of its context.★

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of* P *and a factor not depending on* P.

A-SSE.A.2 Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$, *thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.

Create equations that describe numbers or relationships.

A-CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

A-CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods*.★

A-CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law* $V = IR$ *to highlight resistance* R.★

Understand solving equations as a process of reasoning and explain the reasoning.

A-REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Solve equations and inequalities in one variable.

A-REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Solve systems of equations.

A-REI.C.6[15] Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Represent and solve equations and inequalities graphically.

A-REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

[15] Tasks have a real-world context. In Algebra I, tasks have hallmarks of modeling as a mathematical practice (e.g., less-defined tasks, more of the modeling cycle, etc.).

Focus Standards for Mathematical Practice

MP.1 **Make sense of problems and persevere in solving them.** Students are presented with problems that require them to try special cases and simpler forms of the original problem to gain insight into the problem.

MP.2 **Reason abstractly and quantitatively.** Students analyze graphs of non-constant rate measurements and apply reason (from the shape of the graphs) to infer the quantities being displayed and consider possible units to represent those quantities.

MP.4 **Model with mathematics.** Students have numerous opportunities to solve problems that arise in everyday life, society, and the workplace (e.g., modeling bacteria growth and understanding the federal progressive income tax system).

MP.7 **Look for and make use of structure.** Students reason with and analyze collections of equivalent expressions to see how they are linked through the properties of operations. They discern patterns in sequences of solving equation problems that reveal structures in the equations themselves (e.g., $2x + 4 = 10$, $2(x - 3) + 4 = 10$, $2(3x - 4) + 4 = 10$).

MP.8 **Look for and express regularity in repeated reasoning.** After solving many linear equations in one variable (e.g., $3x + 5 = 8x - 17$), students look for general methods for solving a generic linear equation in one variable by replacing the numbers with letters (e.g., $ax + b = cx + d$). They pay close attention to calculations involving the properties of operations, properties of equality, and properties of inequalities, to find equivalent expressions and solve equations, while recognizing common ways to solve different types of equations.

Terminology

New or Recently Introduced Terms

- **Function** (A *function* is a correspondence between two sets, X and Y, in which each element of X is matched[16] to one and only one element of Y. The set X is called the *domain*; the set Y is called the *range*.)
- **Domain** (Refer to the definition of *function*.)
- **Range** (Refer to the definition of *function*.)
- **Linear Function** (A *linear function* is a polynomial function of degree 1.)
- **Average Rate of Change** (Given a function f whose domain includes the closed interval of real numbers $[a, b]$ and whose range is a subset of the real numbers, the *average rate of change* on the interval $[a, b]$ is $\frac{f(b)-f(a)}{b-a}$.)
- **Piecewise Linear Function** (Given non-overlapping intervals on the real number line, a (real) *piecewise linear function* is a function from the union of the intervals to the set of real numbers such that the function is defined by (possibly different) linear functions on each interval.)

[16] *Matched* can be replaced with *assigned* after students understand that each element of X is matched to exactly one element of Y.

Familiar Terms and Symbols[17]

- Numerical Symbol
- Variable Symbol
- Constant
- Numerical Expression
- Algebraic Expression
- Number Sentence
- Truth Values of a Number Sentence
- Equation
- Solution
- Solution Set
- Simple Expression
- Factored Expression
- Equivalent Expressions
- Polynomial Expression
- Equivalent Polynomial Expressions
- Monomial
- Coefficient of a Monomial
- Terms of a Polynomial

Suggested Tools and Representations

- Coordinate Plane
- Equations and Inequalities
- Graphing Calculator

[17] These are terms and symbols students have seen previously.

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	F-IF.A.1, F-IF.A.2, F-IF.A.3, F-IF.B.4, F-IF.B.5, F-IF.B.6, F-IF.C.7a, F-BF.A.1a, F-LE.A.1, F-LE.A.2, F-LE.A.3
End-of-Module Assessment Task	After Topic D	Constructed response with rubric	A-CED.A.1, A-REI.D.11, A-SSE.B.3c, F-IF.A.1, F-IF.A.2, F-IF.A.3, F-IF.B.4, F-IF.B.6, F-IF.C.7a, F-IF.C.9, F-BF.A.1a, F-BF.B.3, F-LE.A.1, F-LE.A.2, F-LE.A.3, F-LE.B.5