Name ____ Date ______________________

1. State precise definitions of angle, circle, perpendicular, parallel, and line segment based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

   Angle:

   Circle:

   Perpendicular:

   Parallel:

   Line segment:

2. A rigid motion, J, of the plane takes a point, A, as input and gives C as output, i.e., $J(A) = C$. Similarly, $J(B) = D$ for input point B and output point D.

   Jerry claims that knowing nothing else about J, we can be sure that $\overline{AC} \cong \overline{BD}$ because rigid motions preserve distance.

   a. Show that Jerry's claim is incorrect by giving a counterexample (hint: a counterexample would be a specific rigid motion and four points A, B, C, and D in the plane such that the motion takes A to C and B to D, yet $\overline{AC} \ncong \overline{BD}$).

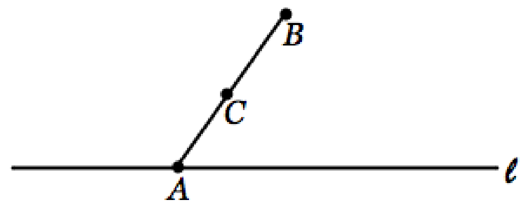
   b. There is a type of rigid motion for which Jerry's claim is always true. Which type below is it?

   Rotation Reflection Translation

   c. Suppose Jerry claimed that $\overline{AB} \cong \overline{CD}$. Would this be true for any rigid motion that satisfies the conditions described in the first paragraph? Why or why not?

3.

a. In the diagram below, l is a line, A is a point on the line, and B is a point not on the line. C is the midpoint of segment $\overline{AB}$. Show how to create a line parallel to l that passes through B by using a rotation about C.



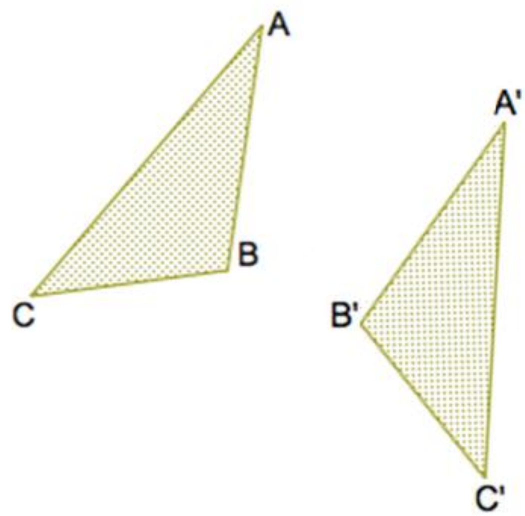
b. Suppose that four lines in a given plane, l_1, l_2, m_1, and m_2 are given, with the conditions (also given) that $l_1 \parallel l_2$, $m_1 \parallel m_2$, and l_1 is neither parallel nor perpendicular to m_1.

i. Sketch (freehand) a diagram of l_1, l_2, m_1, and m_2 to illustrate the given conditions.

ii. In any diagram that illustrates the given conditions, how many distinct angles are formed? Count only angles that measure less than $180°$**,** and count two angles as the same only if they have the same vertex and the same edges. Among these angles, how many different angle <u>measures</u> are formed? Justify your answer.

4. In the figure below, there is a reflection that transforms $\triangle ABC$ to triangle $\triangle A'B'C'$.

   Use a straightedge and compass to construct the line of reflection and list the steps of the construction.



5. Precisely define each of the three rigid motion transformations identified.

   a. $T_{\overrightarrow{AB}}(P)$ ____

   ____

   ____

   ____

   ____

   b. $r_l(P)$ ____

   ____

   ____

   ____

   ____

   c. $R_{C,30^\circ}(P)$ ____

   ____

   ____

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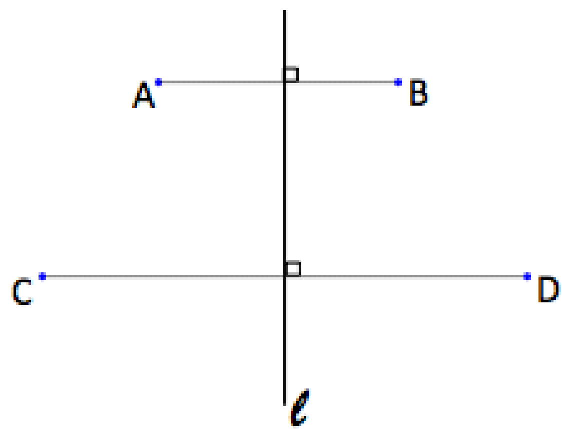
6. Given in the figure below, line l is the perpendicular bisector of $\overline{AB}$ and of $\overline{CD}$.



a. Show $\overline{AC} \cong \overline{BD}$ using rigid motions.

b. Show $\angle ACD \cong \angle BDC$.

c. Show $\overline{AB} \parallel \overline{CD}$.

A Progression Toward Mastery					
Assessment Task Item		**STEP 1** **Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.**	**STEP 2** **Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.**	**STEP 3** **A correct answer with some evidence of reasoning or application of mathematics to solve the problem, <u>or</u> an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.**	**STEP 4** **A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.**
1	**G-CO.A.1**	Student accurately and precisely articulates the definitions of only two of the five terms, but two of the terms are under-developed or poorly defined.	Student accurately and precisely articulates the definitions of at least three of the five terms, but two of the terms are underdeveloped or poorly defined.	Student accurately and precisely articulates the definitions of at least four of the five terms, but one of the terms is underdeveloped or poorly defined.	Student accurately and precisely articulates the definitions of all five terms.
2	**a–c** **G-CO.A.2**	Student circles "translation" in part (b), but the student does not provide a correct response in parts (a) and (b) or provides a response that does not show clear understanding of the application of rigid motions.	Student provides a response that includes a counterexample in part (a) <u>OR</u> presents an idea to prove that $\overline{AB} \cong \overline{CD}$ in part (c). However, whichever is presented is less than perfectly clear in stating the solutions. Student circles "translation" in part (b).	Student provides a counterexample in part (a) and presents an idea to prove that $\overline{AB} \cong \overline{CD}$ in part (c), but both are less than perfectly clear in stating the solutions. Student circles "translation" in part (b).	Student provides a correctly reasoned counterexample in part (a), circles "translation" in part (b), and justifies the claim that $\overline{AB} \cong \overline{CD}$ for any rigid motion in part (c).
3	**a–b** **G-CO.A.1** **G-CO.C.9** **G-CO.D.12**	Student provides an incomplete or irrelevant response in parts (a) and (b.ii) but provides an appropriate, clearly labeled sketch in part (b.i).	Student provides an incomplete description of the rotation of line l about C in part (a), an appropriate, clearly labeled sketch for part (b.i), and an incorrect number of angles formed or an incorrect set of angle measures in part (b.ii).	Student provides an incomplete description of the rotation of line l about C in part (a), an appropriate, clearly labeled sketch for part (b.i), and a justification for why there are 16 relevant angles and 2 different angle measures in part (b.ii).	Student provides a correct description of the rotation of line l about C in part (a), an appropriate, clearly labeled sketch for part (b.i), and a justification for why there are 16 relevant angles and 2 different angle measures in part (b.ii).

4	**G-CO.A.5** **G-CO.D.12**	Student provides a drawing that is not an appropriate construction and an underdeveloped list of steps.	Student provides appropriate construction marks but makes more than one error in the construction or the steps; the line of reflection is drawn.	Student provides appropriate construction marks but makes one error in the construction or the steps; the line of reflection is drawn.	Student draws a correct construction showing all appropriate marks, including the line of reflection, and the accompanying list of steps is also correct.
5	**a–c** **G-CO.A.4**	Student provides inaccurate definitions for the three rigid motions.	Student provides definitions that lack the precise language of an exemplary response, and the student does not address the points that are unchanged (i.e., does not mention that the rotation of the center remains fixed).	Student provides definitions that lack the precise language of an exemplary response.	Student provides precise definitions for each rigid motion with correct usage of notation.
6	**a–c** **G-CO.B.6** **G-CO.C.9**	Student provides an incorrect response or a response that shows little evidence of understanding the properties of reflections.	Student provides an incorrect response, but the response shows evidence of the beginning of understanding of the properties of reflections.	Student provides a response that lacks the precision of an exemplary response, but the response shows an understanding of the properties of reflections.	Student provides a correct response for each of the three parts that demonstrates a clear understanding of the properties of reflections.

Name ____ Date ______________________

1. State precise definitions of angle, circle, perpendicular, parallel, and line segment based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Angle:

An angle is formed by two rays that share a common vertex. An angle is proper if the two rays do not lie on the same line.

Circle:

A circle is the set of all points in a plane that are equidistant from the center point. The circle in plane P with center A and radius AB is the set of all points in P whose distance from A is the same as the distance from A to B.

Perpendicular:

Two lines are perpendicular if they have one point in common and if the four angles formed by the intersection are all right angles.

Parallel:

Two lines are parallel if they lie in the same plane and have no points in common.

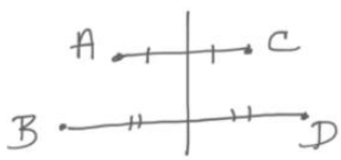
Line segment:

A line segment is the set of point A and B and all points on line AB between A and B.

2. A rigid motion, J, of the plane takes a point, A, as input and gives C as output, i.e., $J(A) = C$. Similarly, $J(B) = D$ for input point B and output point D.

Jerry claims that knowing nothing else about J, we can be sure that $\overline{AC} \cong \overline{BD}$ because rigid motions preserve distance.

a. Show that Jerry's claim is incorrect by giving a counterexample (hint: a counterexample would be a specific rigid motion and four points A, B, C, and D in the plane such that the motion takes A to C and B to D, yet $\overline{AC} \ncong \overline{BD}$).



Here, J is a reflection across a vertical line. The distance from A to the line is different from the distance from B to the line. Therefore, the distance from A to its image (C) is different from the distance from B to its image (D).

b. There is a type of rigid motion for which Jerry's claim is always true. Which type below is it?

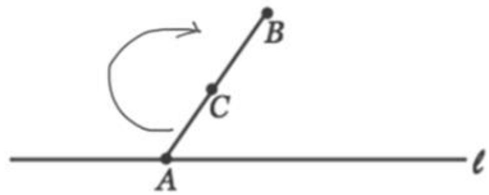
Rotation Reflection Translation

c. Suppose Jerry claimed that $\overline{AB} \cong \overline{CD}$. Would this be true for any rigid motion that satisfies the conditions described in the first paragraph? Why or why not?

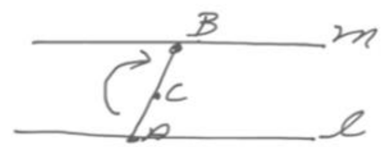
Yes, because rigid motions always preserve distance.

3.

a. In the diagram below, l is a line, A is a point on the line, and B is a point not on the line. C is the midpoint of segment $\overline{AB}$. Show how to create a line parallel to l that passes through B by using a rotation about C.

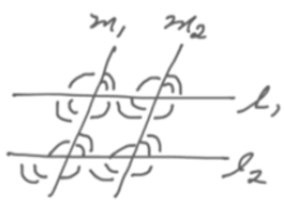


Take a 180° rotation about C. Line m is the image of ℓ under this rotation. Line m passes through B and is parallel to ℓ.



b. Suppose that four lines in a given plane, l_1, l_2, m_1, and m_2 are given, with the conditions (also given) that $l_1 \parallel l_2$, $m_1 \parallel m_2$, and l_1 is neither parallel nor perpendicular to m_1.

i. Sketch (freehand) a diagram of l_1, l_2, m_1, and m_2 to illustrate the given conditions.

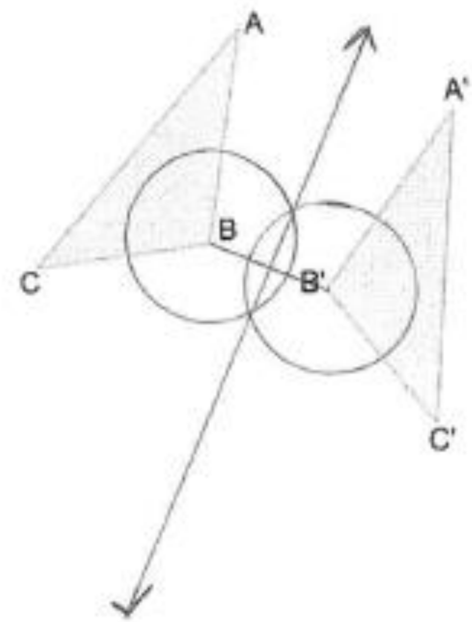


ii. In any diagram that illustrates the given conditions, how many distinct angles are formed? Count only angles that measure less than $180°$**,** and count two angles as the same only if they have the same vertex and the same edges. Among these angles, how many different angle <u>measures</u> are formed? Justify your answer.

There are 16 distinct angles with two different angle measures because alternate interior/exterior angles are congruent and corresponding angles are congruent.

4. In the figure below, there is a reflection that transforms $\triangle ABC$ to triangle $\triangle A'B'C'$.

   Use a straightedge and compass to construct the line of reflection and list the steps of the construction.



1. Draw segment BB'
2. Construct circle B with radius $\overline{BB'}$
3. Construct circle B' with radius $\overline{BB'}$
4. Connect the two intersections of circles B and B'.
5. This forms the line of reflection between ΔABC and $\Delta A'B'C'$

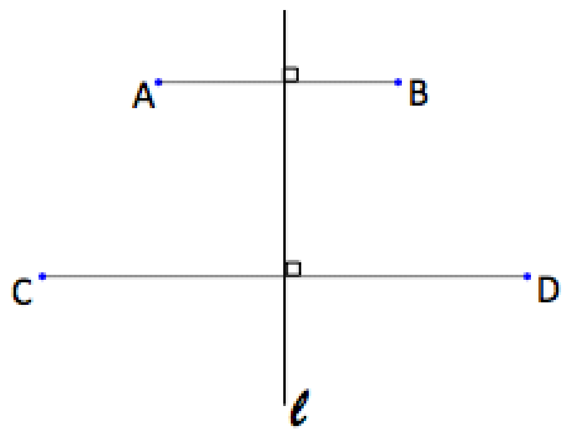
5. Precisely define each of the three rigid motion transformations identified.

a. $T_{\overrightarrow{AB}}(P)$ For vector $\overrightarrow{AB}$, the translation along $\overrightarrow{AB}$ is a translation of the plane: (1) For every point P on line AB, $T_{\overrightarrow{AB}}(P)$ is the point Q on $\overleftrightarrow{AB}$ so that $\overrightarrow{PQ}$ has the same length and direction as $\overrightarrow{AB}$ and (2) For P not on $\overleftrightarrow{AB}$, let ℓ_1 be the line through P parallel to $\overleftrightarrow{AB}$ and ℓ_2 be the line through B parallel to $\overleftrightarrow{AB}$. Q is the intersection of ℓ_1 and ℓ_2.

b. $r_l(P)$ For a line ℓ, a reflection across ℓ is the transformation r_ℓ of the plane defined as follows: (1) for any point P on ℓ, $r_\ell(P) = P$, and (2) For any point P not on ℓ, $r_\ell(P)$ is the point Q so that ℓ is the perpendicular bisector of $\overline{PQ}$

c. $R_{C,30^\circ}(P)$ The rotation of 30° around center C is defined as follows: (1) For the center point C, $R_{C,30^\circ}(C) = C$, and (2) For any other point P, $R_{C,30^\circ}$ is the point Q that lies in the counterclockwise half-plane of ray $\overrightarrow{CP}$, such that $CQ = CP$ and $\angle PCQ = 30^\circ$.

6. Given in the figure below, line l is the perpendicular bisector of $\overline{AB}$ and of $\overline{CD}$.



a. Show $\overline{AC} \cong \overline{BD}$ using rigid motions.

Since l is the perpendicular bisector of $\overline{AB}$ and $\overline{CD}$, the reflection through line l brings A to B and C to D. Because reflections take line segments to congruent line segments, $\overline{AC}$ is congruent to $\overline{BD}$.

b. Show $\angle ACD \cong \angle BDC$.

The reflection through line l brings A to B and C to D and D to C. Therefore ray $\overrightarrow{CA}$ goes to ray $\overrightarrow{DB}$. Ray $\overrightarrow{CD}$ goes to ray $\overrightarrow{DC}$. The image of $\angle ACD$ is therefore congruent to $\angle BDC$.

c. Show $\overline{AB} \parallel \overline{CD}$.

$\overline{AB} \parallel \overline{CD}$ because the perpendicular bisector intersects the two lines creating congruent corresponding angles.