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Congruence, Proof, and Constructions

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Geometry • Module 1

Congruence, Proof, and Constructions

OVERVIEW

Module 1 embodies critical changes in Geometry as outlined by the Common Core. The heart of the module is the study of transformations and the role transformations play in defining congruence.

Students begin this module with Topic A, Basic Constructions. Major constructions include an equilateral triangle, an angle bisector, and a perpendicular bisector. Students synthesize their knowledge of geometric terms with the use of new tools and simultaneously practice precise use of language and efficient communication when they write the steps that accompany each construction (**G.CO.A.1**).

Constructions segue into Topic B, Unknown Angles, which consists of unknown angle problems and proofs. These exercises consolidate students' prior body of geometric facts and prime students' reasoning abilities as they begin to justify each step for a solution to a problem. Students began the proof writing process in Grade 8 when they developed informal arguments to establish select geometric facts (**8.G.A.5**).

Topics C and D, Transformations/Rigid Motions and Congruence, builds on students' intuitive understanding developed in Grade 8. With the help of manipulatives, students observed how reflections, translations, and rotations behave individually and in sequence (**8.G.A.1**, **8.G.A.2**). In high school Geometry, this experience is formalized by clear definitions (**G.CO.A.4**) and more in-depth exploration (**G.CO.A.3**, **G.CO.A.5**). The concrete establishment of rigid motions also allows proofs of facts formerly accepted to be true (**G.CO.C.9**). Similarly, students' Grade 8 concept of congruence transitions from a hands-on understanding (**8.G.A.2**) to a precise, formally notated understanding of congruence (**G.CO.B.6**). With a solid understanding of how transformations form the basis of congruence, students next examine triangle congruence criteria. Part of this examination includes the use of rigid motions to prove how triangle congruence criteria such as SAS actually work (**G.CO.B.7**, **G.CO.B.8**).

In Topic E, Proving Properties of Geometric Figures, students use what they have learned in Topics A through D to prove properties—those that have been accepted as true and those that are new—of parallelograms and triangles (**G.CO.C.10**, **G.CO.C.11**). The module closes with a return to constructions in Topic F (**G.CO.D.13**), followed by a review of the module that highlights how geometric assumptions underpin the facts established thereafter (Topic G).

Focus Standards

Experiment with transformations in the plane.

- G-CO.A.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

- G-CO.A.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
- G-CO.A.3** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
- G-CO.A.4** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
- G-CO.A.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Understand congruence in terms of rigid motions.

- G-CO.B.6** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
- G-CO.B.7** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
- G-CO.B.8** Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Prove geometric theorems.

- G-CO.C.9** Prove² theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*
- G-CO.C.10** Prove² theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*
- G-CO.C.11** Prove² theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

² Prove and apply (in preparation for Regents Exams).

Make geometric constructions.

- G-CO.D.12** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*
- G-CO.D.13** Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Foundational Standards**Understand congruence and similarity using physical models, transparencies, or geometry software.**

- 8.G.A.1** Verify experimentally the properties of rotations, reflections, and translations:
- Lines are taken to lines, and line segments to line segments of the same length.
 - Angles are taken to angles of the same measure.
 - Parallel lines are taken to parallel lines.
- 8.G.A.2** Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
- 8.G.A.3** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
- 8.G.A.5** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

Focus Standards for Mathematical Practice

- MP.3** **Construct viable arguments and critique the reasoning of others.** Students articulate steps needed to construct geometric figures, using relevant vocabulary. Students develop and justify conclusions about unknown angles and defend their arguments with geometric reasons.
- MP.4** **Model with mathematics.** Students apply geometric constructions and knowledge of rigid motions to solve problems arising with issues of design or location of facilities.
- MP.5** **Use appropriate tools strategically.** Students consider and select from a variety of tools in constructing geometric diagrams, including (but not limited to) technological tools.

MP.6 **Attend to precision.** Students precisely define the various rigid motions. Students demonstrate polygon congruence, parallel status, and perpendicular status via formal and informal proofs. In addition, students will clearly and precisely articulate steps in proofs and constructions throughout the module.

Terminology

New or Recently Introduced Terms

- **Isometry** (An *isometry* of the plane is a transformation of the plane that is distance-preserving.)

Familiar Terms and Symbols³

- Transformation
- Translation
- Rotation
- Reflection
- Congruence

Suggested Tools and Representations

- Compass and straightedge
- Geometer’s Sketchpad or Geogebra Software
- Patty paper

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic C	Constructed response with rubric	G-CO.A.1, G-CO.A.2, G-CO.A.4, G-CO.A.5, G-CO.B.6, G-CO.C.9, G-CO.D.12
End-of-Module Assessment Task	After Topic G	Constructed response with rubric	G-CO.A.2, G-CO.A.3, G-CO.B.7, G-CO.B.8, G-CO.C.10, G-CO.C.11, G-CO.D.13

³ These are terms and symbols students have seen previously.