

Name _____

Date _____

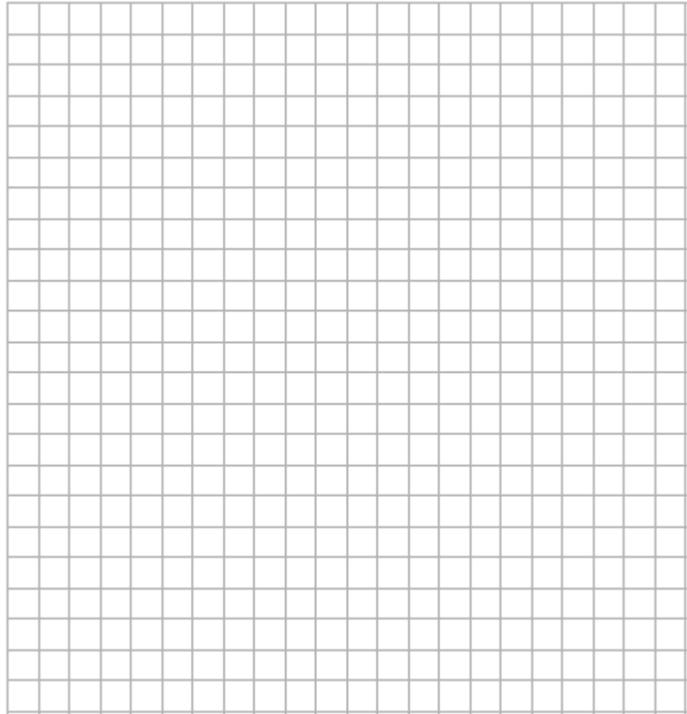
For problems that require rounding, round answers to the nearest hundredth.

1. A triangular region in the coordinate plane is defined by the system of inequalities $y \geq \frac{1}{2}x - 6$, $y \leq -2x + 9$, $y \leq 8x + 9$.

- a. Determine the coordinates of the vertices of the triangle.

- b. Sketch the triangular region defined by these inequalities.

- c. Is the triangle defined by the inequalities a right triangle? Explain your answer.



- d. Find the perimeter of the triangular region defined by the inequalities, round to the nearest hundredth.

- e. What is the area of this triangular region?
- f. Of the three altitudes of the triangular region defined by the inequalities, what is the length of the shortest of the three? Round to the nearest hundredth.
2. Find the point on the directed line segment from $(0,3)$ to $(6,9)$ that divides the segment in the ratio of 2:1.
3. Consider the points $A(1,4)$ and $B(8,-3)$. Suppose C and D are points on the line through A and B satisfying $\frac{AC}{CB} = \frac{1}{3}$ and $\frac{BD}{DA} = \frac{4}{3}$, respectively.
- a. Draw a sketch of the four collinear points A, B, C, D showing their relative positions to one another.

b. Find the coordinates of the point C .

c. Find the coordinates of the point D .

4. Two robots are left in a robotics competition. Robot A is programmed to move about the coordinate plane at a constant speed so that, at time t seconds, its position in the plane is given by

$$(0, 10) + \frac{t}{8}(60, 80).$$

Robot B is also programmed to move about the coordinate plane at a constant speed. Its position in the plane at time t seconds is given by

$$(70, 0) - \frac{t}{10}(70, -70).$$

a. What was each robot's starting position?

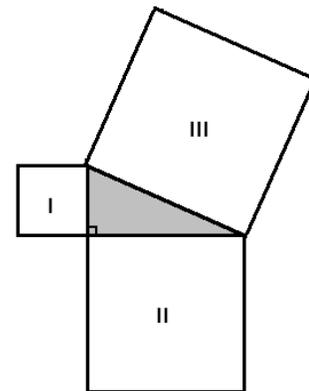
b. Where did each robot stop?

- c. What is the equation of the path of robot A?
- d. What is the equation of the path of robot B?
- e. What is the speed of robot A? (Assume coordinates in the plane are given in units of meters. Give the speed in units of meters per second.)
- f. Do the two robots ever pass through the same point in the plane? Explain. If they do, do they pass through that common point at the same time? Explain.
- g. What is the closest distance robot B will ever be to the origin? Round to the nearest hundredth.

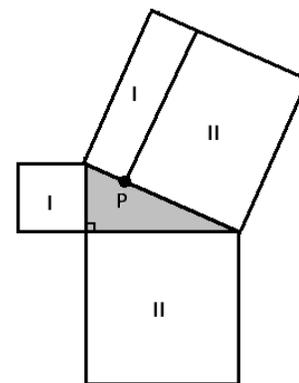
- h. At time $t = 10$ seconds, robot A will instantaneously turn 90 degrees to the left and travel at the same constant speed it was previously traveling. What will be its coordinates in another 10 seconds' time?
5. $GDAY$ is a rhombus. If point G has coordinates $(2, 6)$ and A has coordinates $(8, 10)$, what is the equation of the line that contains the diagonal \overline{DY} of the rhombus?
6. a. A triangle has vertices $A(a_1, a_2)$, $B(b_1, b_2)$, and $C(c_1, c_2)$. Let M be the midpoint of \overline{AC} and N the midpoint of \overline{BC} . Find a general expression for the slope of \overline{MN} . What segment of the triangle has the same slope as \overline{MN} ?

- b. A triangle has vertices $A(a_1, a_2)$, $B(b_1, b_2)$, and $C(c_1, c_2)$. Let P be a point on \overline{AC} with $AP = \frac{5}{8}AC$, and let Q be a point on \overline{BC} with $BQ = \frac{5}{8}BC$. Find a general expression for the slope of \overline{PQ} . What segment of the triangle has the same slope as \overline{PQ} ?
- c. A quadrilateral has vertices $A(a_1, a_2)$, $B(b_1, b_2)$, $C(c_1, c_2)$, and $D(d_1, d_2)$. Let R, S, T , and U be the midpoints of the sides \overline{AB} , \overline{BC} , \overline{CD} , and \overline{DA} , respectively. Demonstrate that \overline{RS} is parallel to \overline{TU} . Is \overline{ST} parallel to \overline{UR} ? Explain.

7. The Pythagorean theorem states that if three squares are drawn on the sides of a right triangle, then the area of the largest square equals the sum of the areas of the two remaining squares.



area III = area I + area II

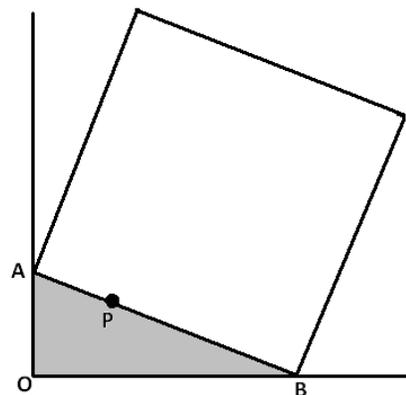


There must be a point P along the hypotenuse of the right triangle at which the large square is divided into two rectangles as shown, each with area matching the area of one of the smaller squares.

Consider a right triangle AOB situated on the coordinate plane with vertex A on the positive y -axis, O the origin, and vertex B on the positive x -axis.

Suppose A has coordinates $(0, a)$, B has coordinates $(b, 0)$, and the length of the hypotenuse \overline{AB} is c .

- a. Find the coordinates of a point P on the line segment \overline{AB} such that \overline{OP} is perpendicular to \overline{AB} .



b. Show that for this point P we have $\frac{AP}{PB} = \frac{a^2}{b^2}$.

c. Show that if we draw from P a line perpendicular to \overline{AB} , then that line divides the square with \overline{AB} as one of its sides into two rectangles, one of area a^2 and one of area b^2 .

A Progression Toward Mastery

Assessment Task Item		STEP 1 Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.	STEP 2 Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.	STEP 3 A correct answer with some evidence of reasoning or application of mathematics to solve the problem, <u>or</u> an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.	STEP 4 A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.
1	a G-GPE.B.5	Student gives no answer or an incorrect answer with no supporting work.	Student equates equations, but no vertices are identified correctly.	Student equated equations and at least one vertex is identified correctly.	Student identifies all vertices correctly.
	b G-GPE.B.5	Student gives no answer or an incorrect answer with no supporting work.	Student sketches at least one inequality correctly.	Student sketches all inequalities correctly but does not shade region.	Students sketches all inequalities correctly and sketches the region.
	c G-GPE.B.4	Student gives no answer or an incorrect answer with no supporting work.	Student attempts to find slopes of lines or length of segments, but errors lead to the wrong answer.	Student uses slope or the converse of the Pythagorean theorem to prove the triangle is right but does not explain the answer.	Student uses slope or the converse of the Pythagorean theorem to prove the triangle is right and explains the answer correctly.
	d G-GPE.B.7	Student gives no answer or an incorrect answer with no supporting work.	Student gives an incorrect answer but work shows use of the distance formula.	Student gives a correct answer with supporting work but not rounded to the hundredths.	Student gives a correct answer with supporting work and properly rounded.
	e G-GPE.B.7	Student gives no answer or an incorrect answer with no supporting work.	Student attempts to use a formula or decomposition to find area but made many errors leading to an incorrect answer.	Student shows knowledge of using a formula or decomposition to find answer but made minor errors leading to an incorrect answer.	Student has correct answer with accurate supporting work.

	f G-GPE.B.5	Student gives no answer or an incorrect answer with no supporting work.	Student shows some knowledge of altitudes of a triangle, identifying the two legs of the right triangle as two altitudes.	Students shows some knowledge of altitudes of a triangle, identifies the two legs of the right triangle, finds the third altitude of 7.68 units, but does not state the shortest altitude.	Students shows some knowledge of altitudes of a triangle, identifies the two legs of the right triangle, finds the third altitude of 7.68 units, and identifies it as the shortest altitude.
2	G-GPE.B.6	Student gives no answer or an incorrect answer with no supporting work.	Student plots the points and shows some knowledge of using an endpoint and the proportion to find the point but answers incorrectly.	Student finds one of the coordinates correctly.	Student finds both coordinates of the point correctly and writes them as an ordered pair.
3	a G-GPE.B.6	Student gives no answer or an incorrect answer with no supporting work.	Student plots points <i>A</i> and <i>B</i> correctly.	Student plots points <i>A</i> and <i>B</i> as endpoints with point <i>C</i> between but point <i>D</i> is closer to <i>A</i> than <i>C</i> .	Student plots all points correctly: <i>A</i> , <i>C</i> , <i>D</i> , <i>B</i> .
	b G-GPE.B.6	Student gives no answer or an incorrect answer with no supporting work.	Student plots the points and shows some knowledge of using an endpoint and the proportion to find the point but answers incorrectly.	Student finds one of the coordinates correctly.	Student finds both coordinates of the point correctly and writes them as an ordered pair.
	c G-GPE.B.6	Student gives no answer or an incorrect answer with no supporting work.	Student plots the points and shows some knowledge of using an endpoint and the proportion to find the point but answers incorrectly.	Student finds one of the coordinates correctly.	Student finds both coordinates of the point correctly and writes them as an ordered pair.
4	a G-GPE.B.6	No answer is given or an incorrect answer with no supporting work.	Student tries to use formula to calculate starting position but neither is correct.	Student finds one of the starting positions correctly.	Student finds both starting positions correctly.
	b G-GPE.B.6	Student gives no answer or an incorrect answer with no supporting work.	Student tries to use formula to calculate where the robot stopped but neither is correct.	Student finds where one of the robots stops correctly.	Student finds where both of the robots stop correctly.
	c G-GPE.B.6	Student gives no answer or an incorrect answer is given with no supporting work.	Student calculates slope correctly but does not attempt to use a point to write the equation of the line.	Student calculates slope correctly and uses a point to write the equation of the line but makes minor errors.	Student calculates the equation of the line of motion correctly.

	d G-GPE.B.6	Student gives no answer or an incorrect answer is given with no supporting work.	Student calculates slope correctly but does not attempt to use a point to write the equation of the line.	Student calculates slope correctly and uses a point to write the equation of the line but makes minor errors.	Student calculates the equation of the line of motion correctly.
	e G-GPE.B.6	Student does not calculate the distance traveled nor the speed or both are calculated incorrectly.	Student calculates the distance correctly but does not divide by time to arrive at speed.	Student calculates speed correctly but units are incorrect or answer is incorrectly rounded.	Student calculates speed correctly with appropriate units and rounding.
	f G-GPE.B.6	Student gives no answer or an incorrect answer with no supporting work.	Student equates line to find the intersection point but makes mistakes in calculations.	Student finds the correct intersection point but does not find the time that each robot crosses that point.	Student finds the correct intersection point and explains that the robots will not cross at the same time justifying with times.
	g G-GPE.B.5	Student gives no answer or an incorrect answer with no supporting work.	Student uses the equation of motion of robot B but does not use the perpendicular segment to the line through the origin.	Student uses the equation of motion of robot B and the perpendicular line through the origin and find the closest point (35, 35) but not the distance.	Student uses the equation of motion of robot B and the perpendicular line through the origin and finds the distance from (35, 35) to the origin.
	h G-GPE.B.5	Student gives no answer or an incorrect answer with no supporting work.	Student finds the position at 10 seconds (75, 110) but does not move the robot past that point.	Student finds the position at 10 seconds (75,100) and rotates the motion but calculates the final position incorrectly.	Student finds the position at 10 seconds and rotates and moves the robot to the correct final position (-35, 185).
5	G-GPE.B.4 G-GPE.B.5	No answer is given or an incorrect answer with no supporting work.	Student shows knowledge of the properties of a rhombus and finds the slope of a line perpendicular to the segment given.	Student shows knowledge of the properties of a rhombus and finds the slope of the line perpendicular to the segment given and the midpoint but does not write the equation of the line.	Student uses the given perpendicular slope and midpoint of segment to write an equation of the other diagonal.
6	a G-GPE.B.6	No answer is given or an incorrect answer with no supporting work.	Student shows knowledge of the midpoint formula, but errors lead to an incorrect answer.	Student finds the midpoints M and N correctly but does not find slope or slope is incorrect.	Student finds the correct slope and sees that it is the same slope as \overline{AB} .

	b G-GPE.B.6	Student gives no answer or an incorrect answer with no supporting work.	Student shows knowledge points on directed segments and finds the coordinates of that point, but errors lead to an incorrect answer.	Student finds the point P and Q correctly but does not find slope or slope is incorrect.	Student finds the correct slope and sees that it is the same slope as \overline{AB} .
	c G-GPE.B.5	Student gives no answer or an incorrect answer with no supporting work.	Student finds the slopes of pairs of segments, but mistakes lead to incorrect answers.	Student finds the slopes of pairs of segments correctly but does not compare slopes and explain answers.	Student finds the slopes of pairs of segments correctly and explains why segments are parallel using slopes.
7	a G-GPE.B.5	Student gives no answer or an incorrect answer with no supporting work.	Student writes the equation of \overline{AB} correctly.	Student writes the equation of \overline{AB} and \overline{OP} but does not find point P or finds it incorrectly.	Student writes the equation of \overline{AB} and \overline{OP} and finds point P correctly.
	b G-GPE.B.4	Student gives no answer or an incorrect answer with no supporting work.	Student finds either AP or PB correctly.	Student finds AP and PB correctly and attempts to simplify the ratio but errors lead to incorrect answers.	Student finds AP and PB , writes them as a ratio, and simplifies the ratio correctly.
	c G-GPE.B.4	Student gives no answer or an incorrect answer with no supporting work.	Student shows knowledge that one dimension of rectangle I and II is $a^2 + b^2$.	Student shows that one dimension of rectangle I and II is $a^2 + b^2$ and tries to use the other dimension but writes the wrong ratio.	Student shows that one dimension of rectangle I and II is $a^2 + b^2$, writes the other dimension as the correct ratio, and finds the correct areas.

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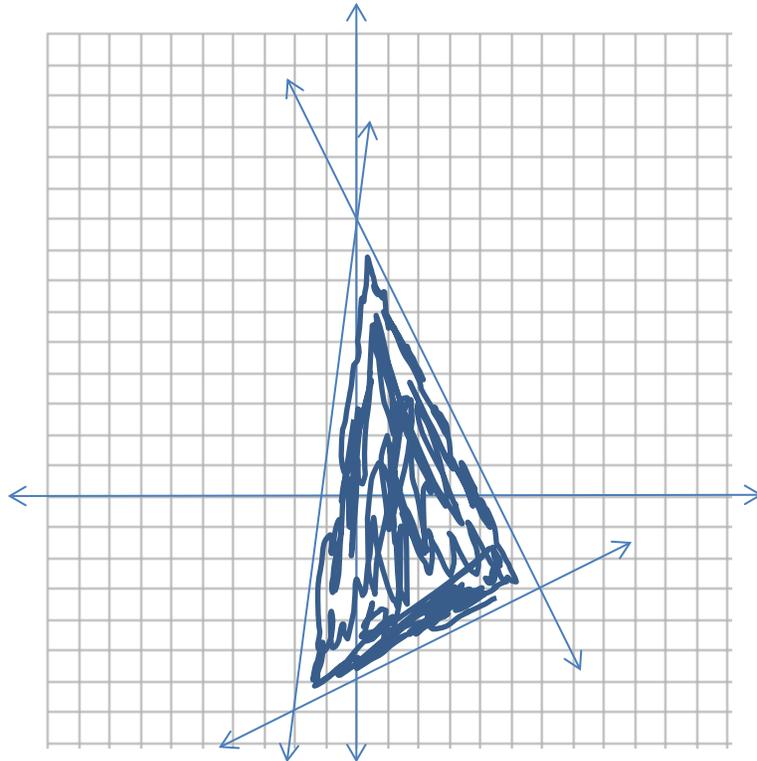
For problems that require rounding, round answers to the nearest hundredth.

1. A triangular region in the coordinate plane is defined by the system of inequalities
 $y \geq \frac{1}{2}x - 6$, $y \leq -2x + 9$, $y \leq 8x + 9$.

- a. Determine the coordinates of the vertices of the triangle.

$(0, 9), (6, -3), (-2, -7)$

- b. Sketch the triangular region defined by these inequalities.



- c. Is the triangle defined by the inequalities a right triangle? Explain your answer.

Yes, slopes of \overline{AB} and \overline{BC} are negative reciprocals, so they are perpendicular.

$AB^2 + BC^2 = AC^2$; therefore, the converse of the Pythagorean theorem is true meaning it is a right triangle.

- d. Find the perimeter of the triangular region defined by the inequalities, round to the nearest hundredth.

38.49 units

- e. What is the area of this triangular region?

60 square units

- f. Of the three altitudes of the triangular region defined by the inequalities, what is the length of the shortest of the three? Round to the nearest hundredth.

Altitudes: from $\angle A = 13.42$, from $\angle C = 8.94$, from $\angle B = 7.68$ (shortest)

2. Find the point on the directed line segment from $(0,3)$ to $(6,9)$ that divides the segment in the ratio of 2:1.

(4, 7)

3. Consider the points $A(1,4)$ and $B(8,-3)$. Suppose C and D are points on the line through A and B satisfying $\frac{AC}{CB} = \frac{1}{3}$ and $\frac{BD}{DA} = \frac{4}{3}$, respectively.

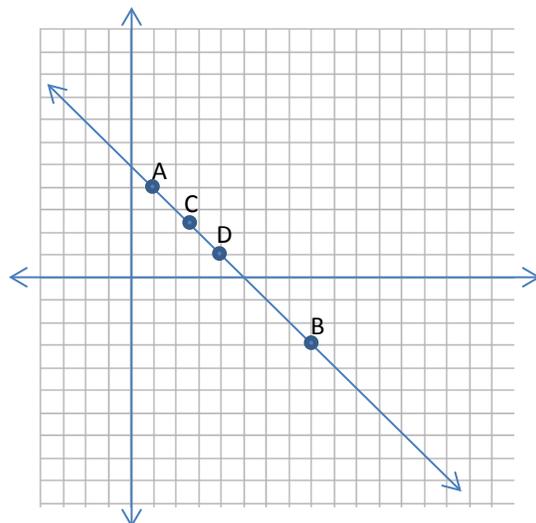
- a. Draw a sketch of the four collinear points A, B, C, D showing their relative positions to one another.

- b. Find the coordinates of the point C .

$(\frac{11}{4}, \frac{9}{4})$

- c. Find the coordinates of the point D .

(4, 1)



4. Two robots are left in a robotics competition. Robot A is programmed to move about the coordinate plane at a constant speed so that, at time t seconds, its position in the plane is given by

$$(0, 10) + \frac{t}{8}(60, 80).$$

Robot B is also programmed to move about the coordinate plane at a constant speed. Its position in the plane at time t seconds is given by

$$(70, 0) - \frac{t}{10}(70, -70).$$

- a. What was each robot's starting position?

$$A(0, 10), B(70, 0)$$

- b. Where did each robot stop?

$$A(60, 90), B(0, 70)$$

- c. What is the equation of the path of robot A?

$$y = \frac{4}{3}x + 10$$

- d. What is the equation of the path of robot B?

$$y = -x + 70$$

- e. What is that speed of robot A? (Assume coordinates in the plane are given in units of meters. Give the speed in units of meters per second.)

$$\text{Distance} = 100 \text{ m, time} = 8 \text{ seconds, speed} = 12.5 \text{ m/sec}$$

- f. Do the two robots ever pass through the same point in the plane? Explain. If they do, do they pass through that common point at the same time? Explain.

The robots both pass through the point $(28\frac{2}{3}, 48\frac{2}{9})$. A passes through that point at 3.82 seconds and B at 6.89 seconds. So the robots do not pass through the point at the same time.

- g. What is the closest distance robot B will ever be to the origin? Round to the nearest hundredth.

The closest point that the robot will be to the origin is $(35, 35)$ and that is a distance of 49.50 meters.

- h. At time $t = 10$ seconds, robot A will instantaneously turn 90 degrees to the left and travel at the same constant speed it was previously traveling. What will be its coordinates in another 10 seconds' time?

At 10 seconds, robot A will be at $(75, 110)$. After turning 90° and continuing for another 10 seconds, the robot will be at $(-35, 185)$.

5. $GDAY$ is a rhombus. If point G has coordinates $(2, 6)$ and A has coordinates $(8, 10)$, what is the equation of the line that contains the diagonal \overline{DY} of the rhombus?

Slope of \overline{GA} is $\frac{2}{3}$, the diagonal will be perpendicular to \overline{GA} and bisect it, so the slope of \overline{DY} is $-\frac{3}{2}$ and it passes through the point $(5, 8)$. The equation of the line is $3x + 2y = 31$.

6. a. A triangle has vertices $A(a_1, a_2)$, $B(b_1, b_2)$, and $C(c_1, c_2)$. Let M be the midpoint of \overline{AC} and N the midpoint of \overline{BC} . Find a general expression for the slope of \overline{MN} . What segment of the triangle has the same slope as \overline{MN} ?

$$M \left(\frac{a_1 + c_1}{2}, \frac{a_2 + c_2}{2} \right), N \left(\frac{b_1 + c_1}{2}, \frac{b_2 + c_2}{2} \right)$$

$$\text{Slope } \overline{MN} = \frac{\frac{b_2 + c_2}{2} - \frac{a_2 + c_2}{2}}{\frac{b_1 + c_1}{2} - \frac{a_1 + c_1}{2}} = \frac{b_2 - a_2}{b_1 - a_1}$$

Slope \overline{MN} is the same as slope of \overline{AB} .

- b. A triangle has vertices $A(a_1, a_2)$, $B(b_1, b_2)$, and $C(c_1, c_2)$. Let P be a point on \overline{AC} with $AP = \frac{5}{8}AC$, and let Q be a point on \overline{BC} with $BQ = \frac{5}{8}BC$. Find a general expression for the slope of \overline{PQ} . What segment of the triangle has the same slope as \overline{PQ} ?

$$P \left(a_1 + \frac{5}{8}(c_1 - a_1), a_2 + \frac{5}{8}(c_2 - a_2) \right), Q \left(b_1 + \frac{5}{8}(c_1 - b_1), b_2 + \frac{5}{8}(c_2 - b_2) \right)$$

$$\text{Slope } \overline{PQ} = \frac{\frac{3b_2}{8} - \frac{3a_2}{8}}{\frac{3b_1}{8} - \frac{3a_1}{8}} = \frac{b_2 - a_2}{b_1 - a_1}$$

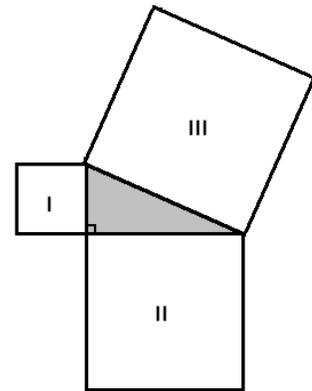
Slope \overline{PQ} is the same as slope of \overline{AB} .

- c. A quadrilateral has vertices $A(a_1, a_2)$, $B(b_1, b_2)$, $C(c_1, c_2)$, and $D(d_1, d_2)$. Let $R, S, T,$ and U be the midpoints of the sides \overline{AB} , \overline{BC} , \overline{CD} , and \overline{DA} , respectively. Demonstrate that \overline{RS} is parallel to \overline{TU} . Is \overline{ST} parallel to \overline{UR} ? Explain.

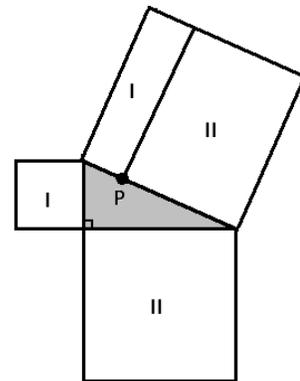
$$\text{Slope } \overline{RS} = \frac{\frac{b_2 + c_2}{2} - \frac{a_2 + b_2}{2}}{\frac{b_1 + c_1}{2} - \frac{a_1 + b_1}{2}} = \frac{c_2 - a_2}{c_1 - a_1} = \text{slope } \overline{TU} \text{ since slopes are equal, segments are parallel.}$$

$$\text{Slope } \overline{ST} = \frac{\frac{b_2 + c_2}{2} - \frac{c_2 + d_2}{2}}{\frac{b_1 + c_1}{2} - \frac{c_1 + d_1}{2}} = \frac{b_2 - d_2}{b_1 - d_1} = \text{slope } \overline{UR} \text{ since slopes are equal, segments are parallel.}$$

7. The Pythagorean theorem states that if three squares are drawn on the sides of a right triangle, then the area of the largest square equals the sum of the areas of the two remaining squares.



There must be a point P along the hypotenuse of the right triangle at which the large square is divided into two rectangles as shown, each with area matching the area of one of the smaller squares.



Consider a right triangle AOB situated on the coordinate plane with vertex A on the positive y -axis, O the origin, and vertex B on the positive x -axis.

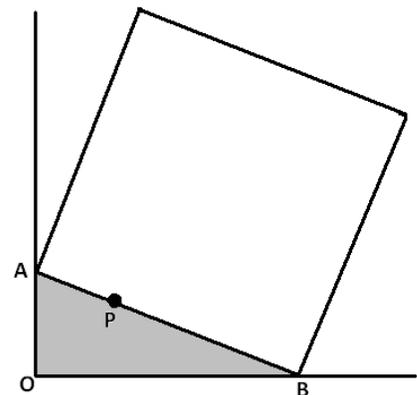
Suppose A has coordinates $(0, a)$, B has coordinates $(b, 0)$, and the length of the hypotenuse \overline{AB} is c .

- a. Find the coordinates of a point P on the line segment \overline{AB} such that \overline{OP} is perpendicular to \overline{AB} .

$$\text{Equation of } \overline{AB}: y = -\frac{a}{b}x + a$$

$$\text{Equation of } \overline{OP}: y = \frac{b}{a}x$$

$$P \left(\frac{a^2b}{a^2+b^2}, \frac{ab^2}{a^2+b^2} \right)$$



- b. Show that for this point P we have $\frac{AP}{PB} = \frac{a^2}{b^2}$.

$$AP = \sqrt{\left(\frac{a^2b}{a^2+b^2}\right)^2 + \left(\frac{ab^2}{a^2+b^2} - a\right)^2}, \quad PB = \sqrt{\left(b - \frac{a^2b}{a^2+b^2}\right)^2 + \left(\frac{ab^2}{a^2+b^2}\right)^2}$$

$$\frac{AP}{PB} = \sqrt{\frac{a^4b^2 + b^6}{b^6 + a^2b^4}} = \sqrt{\frac{a^4(b^2 + a^2)}{b^4(b^2 + a^2)}} = \sqrt{\frac{a^4}{b^4}} = \frac{a^2}{b^2}$$

- c. Show that if we draw from P a line perpendicular to \overline{AB} , then that line divides the square with \overline{AB} as one of its sides into two rectangles, one of area a^2 and one of area b^2 .

$$\text{Rectangle I: width} = a^2 + b^2, \text{ height} = \frac{a^2}{a^2 + b^2}, \text{ area} = a^2$$

$$\text{Rectangle II: width} = a^2 + b^2, \text{ height} = \frac{b^2}{a^2 + b^2}, \text{ area} = b^2$$