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Connecting Algebra and Geometry Through Coordinates

[1] Each lesson is ONE day, and ONE day is considered a 45-minute period.

Geometry • Module 4

Connecting Algebra and Geometry Through Coordinates

OVERVIEW

In this module, students explore and experience the utility of analyzing algebra and geometry challenges through the framework of coordinates. The module opens with a modeling challenge (**G-MG.A.1**, **G-MG.A.3**), one that reoccurs throughout the lessons, to use coordinate geometry to program the motion of a robot that is bound within a certain polygonal region of the plane—the room in which it sits. To set the stage for complex work in analytic geometry (computing coordinates of points of intersection of lines and line segments or the coordinates of points that divide given segments in specific length ratios, and so on), students will describe the region via systems of algebraic inequalities (**A-REI.D.12**) and work to constrain the robot motion along line segments within the region (**A-REI.C.6**, **G-GPE.B.7**).

The challenge of programming robot motion along segments parallel or perpendicular to a given segment brings in an analysis of slopes of parallel and perpendicular lines and the need to prove results about these quantities (**G-GPE.B.5**). This work highlights the role of the converse of the Pythagorean theorem in the identification of perpendicular directions of motion (**G-GPE.B.4**).

To fully develop the analysis of perimeter and area of a polygon in terms of the coordinates of its vertices (**G-GPE.B.7**), students will derive the area A of a triangle with coordinates $(0,0)$, (x_1, y_2), and (x_2, y_2) as $A = \left(\frac{1}{2}\right)|x_1y_1 - x_2y_2|$ and extend this result to the areas of triangles situated elsewhere in the plane and to simple polygons seen as unions of triangles. Applications to robot motion continue. Students will also find locations on a directed line segment between two given points that partition the segment in given ratios (**G-GPE.B.6**) and connect this work to proving classical results in geometry (**G-GPE.B.4**). For example, proving that the diagonals of a parallelogram bisect one another, and the medians of a triangle meet at the point $\frac{2}{3}$ of the way from the vertex for each. This study also deepens student understanding of the linear motion of the robot between and beyond two given points.

The module ends with the challenge of locating the point along a line closest to a given point, again given as a robot challenge, and developing the distance formula for a point from a line (**G-GPE.B.4**).

Focus Standards

Use coordinates to prove simple geometric theorems algebraically.[2]

G-GPE.B.4 Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point* $(1, \sqrt{3})$ *lies on the circle centered at the origin and containing the point* $(0, 2)$.

G-GPE.B.5 Prove[3] the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

G-GPE.B.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

G-GPE.B.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★

Foundational Standards

Solve systems of equations.

A-REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Represent and solve equations and inequalities graphically.

A-REI.D.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Apply geometric concepts in modeling situations.

G-MG.A.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★

G-MG.A.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★

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[2] In preparation for Regents Exams, a trapezoid is defined as a quadrilateral that has at least one pair of parallel sides.

[3] Prove *and apply* (in preparation for Regents Exams).

Focus Standards for Mathematical Practice

MP.1 **Make sense of problems and persevere in solving them.** Students start the module with the challenge to understand and develop the mathematics for describing of the motion of a robot bound within a certain polygonal region of the plane—the room in which it sits. This a recurring problem throughout the entire module and with it, and through related problems, students discover the slope criteria for perpendicular and parallel lines, the means to find the coordinates of a point dividing a line segment into two lengths in a given ratio, the distance formula of a point from a line, along with a number of geometric results via the tools of coordinate geometry.

MP.2 **Reason abstractly and quantitatively.** Students rotate line segments about their endpoints and discover the general slope criterion for perpendicular lines and articulate this criterion in an abstract setting. Geometric results (such as "the three medians of a triangle are concurrent") are examined in concrete settings and students determine that these results hold in general. They also develop a formula for the area of a triangle based solely on the coordinates of its three vertices and generalize this to an area formula for quadrilaterals and other planar polygons.

MP.4 **Model with mathematics.** Students model the motion of a robot in the plane in two contexts: determining the extent of motion within the bounds of a polygonal region, and determining and moving to the location of the source of beacon signal in the infinite plane.

MP.7 **Look for and make use of structure.** Students determine slope criteria for perpendicular and parallel lines and use these slope conditions to develop the general equation of a line and the formula for the distance of a point from a line. Students determine the area of polygonal regions using multiple methods including Green's theorem and decomposition. Definitive geometric properties of special quadrilaterals are explored and properties of special lines in triangles are examined.

MP.8 **Look for and express regularity in repeated reasoning.** Students use the midpoint to repeatedly separate a segment into proportional parts and derive a formula for calculating the coordinates of a point that will divide a segment into segments of given ratios.

Terminology

New or Recently Introduced Terms

- **Normal Segment to a Line** (A line segment with one endpoint on a line and perpendicular to the line is called a *normal segment* to the line.)

Familiar Terms and Symbols[4]

- Slope
- Parallel
- Perpendicular
- Distance
- Bisect
- Directed Line Segment

Suggested Tools and Representations

- Graph Paper
- Graphing Calculator
- Wolfram Alpha Software
- Geometer's Sketchpad Software

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	G-GPE.B.4, G-GPE.B.5, G-GPE.B.7
End-of-Module Assessment Task	After Topic D	Constructed response with rubric	G-GPE.B.4, G-GPE.B.5, G-GPE.B.6, G-GPE.B.7

[4] These are terms and symbols students have seen previously.