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Ratios and Unit Rates

[1] Each lesson is ONE day, and ONE day is considered a 45-minute period.

Grade 6 • Module 1

Ratios and Unit Rates

OVERVIEW

In this module, students are introduced to the concepts of ratio and rate. Their previous experience solving problems involving multiplicative comparisons, such as *"Max has three times as many toy cars as Jack,"* (**4.OA.A.2**) serves as the conceptual foundation for understanding ratios as a multiplicative comparison of two or more numbers used in quantities or measurements (**6.RP.A.1**). Students develop fluidity in using multiple forms of ratio language and ratio notation. They construct viable arguments and communicate reasoning about ratio equivalence as they solve ratio problems in real world contexts (**6.RP.A.3**). As the first topic comes to a close, students develop a precise definition of the value of a ratio $a: b$, where $b \neq 0$ as the value $\frac{a}{b}$, applying previous understanding of fraction as division (**5.NF.B.3**). They can then formalize their understanding of equivalent ratios as ratios having the same value.

With the concept of ratio equivalence formally defined, students explore collections of equivalent ratios in real world contexts in Topic B. They build ratio tables and study their additive and multiplicative structure (**6.RP.A.3a**). Students continue to apply reasoning to solve ratio problems while they explore representations of collections of equivalent ratios and relate those representations to the ratio table (**6.RP.A.3**). Building on their experience with number lines, students represent collections of equivalent ratios with a double number line model. They relate ratio tables to equations using the value of a ratio defined in Topic A. Finally, students expand their experience with the coordinate plane (**5.G.A.1**, **5.G.A.2**) as they represent collections of equivalent ratios by plotting the pairs of values on the coordinate plane. The Mid-Module Assessment follows Topic B.

In Topic C, students build further on their understanding of ratios and the value of a ratio as they come to understand that a ratio of 5 miles to 2 hours corresponds to a rate of 2.5 miles per hour, where the *unit rate* is the numerical part of the rate, 2.5, and *miles per hour* is the newly formed unit of measurement of the rate (**6.RP.A.2**). Students solve unit rate problems involving unit pricing, constant speed, and constant rates of work (**6.RP.A.3b**). They apply their understanding of rates to situations in the real world. Students determine unit prices and use measurement conversions to comparison shop, and decontextualize constant speed and work situations to determine outcomes. Students combine their new understanding of rate to connect and revisit concepts of converting among different-sized standard measurement units (**5.MD.A.1**). They then expand upon this background as they learn to manipulate and transform units when multiplying and dividing quantities (**6.RP.A.3d**). Topic C culminates as students interpret and model real-world scenarios through the use of unit rates and conversions.

In the final topic of the module, students are introduced to percent and find percent of a quantity as a *rate per* 100. Students understand that N percent of a quantity has the same value as $\frac{N}{100}$ of that quantity. Students express a fraction as a percent, and find a percent of a quantity in real-world contexts. Students learn to express a ratio using the language of percent and to solve percent problems by selecting from familiar representations, such as tape diagrams and double number lines, or a combination of both (**6.RP.A.3c**). The End-of-Module Assessment follows Topic D.

Focus Standards

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.A.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*

6.RP.A.2 Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with *b ≠ 0*, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[2]

6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*

c. Find a percent of a quantity as a rate per *100* (e.g., *30%* of a quantity means *30/100* times the quantity); solve problems involving finding the whole, given a part and the percent.

d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Foundational Standards

Use the four operations with whole numbers to solve problems.

4.OA.A.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[3]

[2] Expectations for unit rates in this grade are limited to non-complex fractions.
[3] See Glossary, Table 2.

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.B.3 Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

Convert like measurement units within a given measurement system.

5.MD.A.1 Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

Graph points on the coordinate plane to solve real-world and mathematical problems.

5.G.A.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).

5.G.A.2 Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Focus Standards for Mathematical Practice

MP.1 **Make sense of problems and persevere in solving them.** Students make sense of and solve real-world and mathematical ratio, rate, and percent problems using representations, such as tape diagrams, ratio tables, the coordinate plane, and double number line diagrams. They identify and explain the correspondences between the verbal descriptions and their representations and articulate how the representation depicts the relationship of the quantities in the problem. Problems include ratio problems involving the comparison of three quantities, multistep changing ratio problems, using a given ratio to find associated ratios, and constant rate problems including two or more people or machines working together.

MP.2 **Reason abstractly and quantitatively.** Students solve problems by analyzing and comparing ratios and unit rates given in tables, equations, and graphs. Students decontextualize a given constant speed situation, representing symbolically the quantities involved with the formula, *distance = rate x time*.

MP.5 **Use appropriate tools strategically.** Students become proficient using a variety of representations that are useful in reasoning with rate and ratio problems, such as tape diagrams, double line diagrams, ratio tables, a coordinate plane, and equations. They then use judgment in selecting appropriate tools as they solve ratio and rate problems.

MP.6 **Attend to precision.** Students define and distinguish between ratio, the value of a ratio, a unit rate, a rate unit, and a rate. Students use precise language and symbols to describe ratios and rates. Students learn and apply the precise definition of percent.

MP.7 **Look for and make use of structure.** Students recognize the structure of equivalent ratios in solving word problems using tape diagrams. Students identify the structure of a ratio table and use it to find missing values in the table. Students make use of the structure of division and ratios to model 5 *miles*/2 *hours* as a quantity 2.5 *mph*.

Terminology

New or Recently Introduced Terms

- **Ratio** (A pair of nonnegative numbers, $A: B$, where both are not zero, and that are used to indicate that there is a relationship between two quantities such that when there are A units of one quantity, there are B units of the second quantity.)
- **Rate** (A rate indicates, for a proportional relationship between two quantities, how many units of one quantity there are for every 1 unit of the second quantity. For a ratio of $A: B$ between two quantities, the rate is A/B units of the first quantity per unit of the second quantity.)
- **Unit Rate** (The numeric value of the rate, e.g., in the rate 2.5 mph, the unit rate is 2.5.)
- **Value of a Ratio** (For the ratio $A: B$, the value of the ratio is the quotient A/B.)
- **Equivalent Ratios** (Ratios that have the same value.)
- **Percent** (Percent of a quantity is a rate per 100.)
- **Associated Ratios** (e.g., if a popular shade of purple is made by mixing 2 cups of blue paint for every 3 cups of red paint, not only can we say that the ratio of blue paint to red paint in the mixture is $2: 3$, but we can discuss associated ratios such as the ratio of cups of red paint to cups of blue paint, the ratio of cups of blue paint to total cups of purple paint, the ratio of cups of red paint to total cups of purple paint, etc.)
- **Double Number Line** (See example under Suggested Tools and Representations.)
- **Ratio Table** (A table listing pairs of numbers that form equivalent ratios; see example under Suggested Tools and Representations.)

Familiar Terms and Symbols[4]

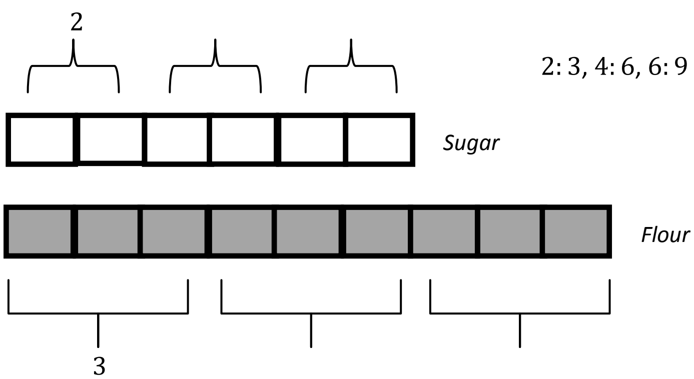
- Convert
- Tape Diagram
- Coordinate Plane
- Equation

Suggested Tools and Representations

- Tape Diagrams (See example below.)
- Double Number Line Diagrams (See example below.)
- Ratio Tables (See example below.)
- Coordinate Plane (See example below.)

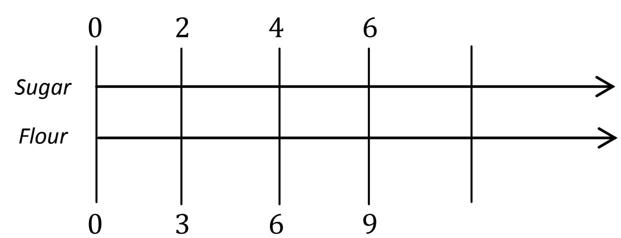
Representing Equivalent Ratios for a cake recipe
that uses 2 cups of sugar for every 3 cups of flour

Tape Diagram



$2:3$, $4:6$, $6:9$

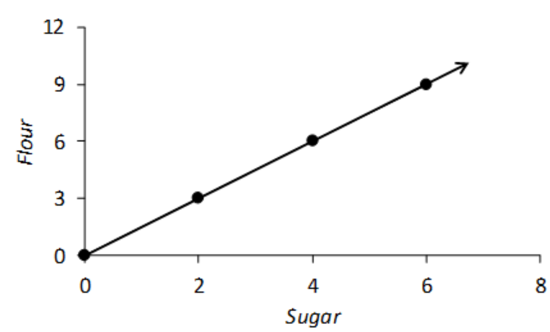
Double Number Line



Ratio Table

Sugar	*Flour*
2	3
4	6
6	9

Coordinate Plane



[4] These are terms and symbols students have seen previously.

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	6.RP.A.1, 6.RP.A.3 (Stem Only), 6.RP.A.3a
End-of-Module Assessment Task	After Topic D	Constructed response with rubric	6.RP.A.1, 6.RP.A.2, 6.RP.A.3