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Area, Surface Area, and Volume Problems

[1] Each lesson is ONE day, and ONE day is considered a 45-minute period.

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Topics C through D (assessment 1 day, return 1 day, remediation or further applications 1 day)

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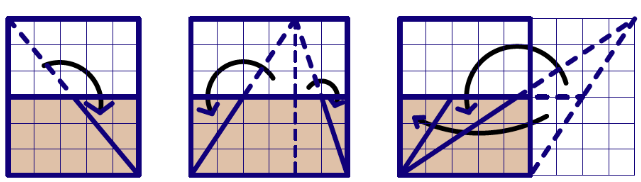
Grade 6 • Module 5

Area, Surface Area, and Volume Problems

OVERVIEW

Starting in Grade 1, students compose and decompose plane and solid figures (**1.G.A.2**). They move to spatial structuring of rectangular arrays in Grade 2 (**2.G.A.2**) and continually build upon their understanding of arrays to ultimately apply their knowledge to two- and three-dimensional figures in Grade 4 (**4.MD.A.3**) and Grade 5 (**5.MD.C.3**, **5.MD.C.5**). Students move from building arrays to using arrays to find area and eventually move to decomposing three-dimensional shapes into layers that are arrays of cubes. In this module, students utilize their previous experiences in shape composition and decomposition in order to understand and develop formulas for area, volume, and surface area.

In Topic A, students use composition and decomposition to determine the area of triangles, quadrilaterals, and other polygons. They determine that area is additive. Students learn through exploration that the area of a triangle is exactly half of the area of its corresponding rectangle. In Lesson 1, students discover through composition that the area of a parallelogram is the same as a rectangle. In Lesson 2, students compose rectangles using two copies of a right triangle. They extend their previous knowledge about the area formula for rectangles (**4.MD.A.3**) to evaluate the area of the rectangle using $A = bh$ and discover through manipulation that the area of a right triangle is exactly half that of its corresponding rectangle. In Lesson 3, students discover that any triangle may be decomposed into right triangles, and in Lesson 4, students further explore all triangles and discover through manipulation that the area of all triangles is exactly half the area of its corresponding rectangle. During this discovery process, students become aware that triangles have altitude, which is the length of the height of the triangle. The altitude is the perpendicular segment from a vertex of a triangle to the line containing the opposite side. The opposite side is called the base. Students understand that any side of the triangle can be a base, but the altitude always determines the base. They move from recognizing right triangles as categories (**4.G.A.2**) to determining that right triangles are constructed when altitudes are perpendicular and meet the base at one endpoint. Acute triangles are constructed when the altitude is perpendicular and meets within the length of the base, and obtuse triangles are constructed when the altitude is perpendicular and lies outside the length of the base. Students use this information to cut triangular pieces and rearrange them to fit exactly within one half of the corresponding rectangle to determine that the area formula for any triangle can be determined using $A = \frac{1}{2}bh$.



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In Lesson 5, students apply their knowledge of the area of a triangular region, where they deconstruct parallelograms, trapezoids, and other quadrilaterals and polygons into triangles or rectangles in order to determine area. They intuitively decompose rectangles to determine the area of polygons. Topic A closes with Lesson 6 where students apply their learning from the topic to find areas of composite figures in real-life contexts, as well as to determine the area of missing regions (**6.G.A.1**).

In Module 3, students used coordinates and absolute value to find distances between points on a coordinate plane (**6.NS.C.8**). In Topic B, students extend this learning to Lessons 7 and 8 where they find edge lengths of polygons (the distance between two vertices using absolute value) and draw polygons given coordinates (**6.G.A.3**). From these drawings, students determine the area of polygons on the coordinate plane by composing and decomposing into polygons with known area formulas. In Lesson 9, students further investigate and calculate the area of polygons on the coordinate plane and also calculate the perimeter. They note that finding perimeter is simply finding the sum of the polygon's edge lengths (or finding the sum of the distances between vertices). Topic B concludes with students determining distance, perimeter, and area on the coordinate plane in real-world contexts.

In Grade 5, students recognized volume as an attribute of solid figures. They measured volume by packing right rectangular prisms with unit cubes and found that determining volume was the same as multiplying the edge lengths of the prism (**5.MD.C.3**, **5.MD.C.4**). Students extend this knowledge to Topic C where they continue packing right rectangular prisms with unit cubes; however, this time the right rectangular prism has fractional lengths (**6.G.A.2**). In Lesson 11, students decompose a one cubic unit prism in order to conceptualize finding the volume of a right rectangular prism with fractional edge lengths using unit cubes. They connect those findings to apply the formula $V = lwh$ and multiply fractional edge lengths (**5.NF.B.4**). In Lessons 12 and 13, students extend and apply the volume formula to $V = \text{The area of the base} \times \text{height}$ or simply $V = bh$, where b represents the area of the base. In Lesson 12, students explore the bases of right rectangular prisms and find the area of the base first, then multiply by the height. They determine that two formulas can be used to find the volume of a right rectangular prism. In Lesson 13, students apply both formulas to application problems. Topic C concludes with real-life application of the volume formula where students extend the notion that volume is additive (**5.MD.C.5c**) and find the volume of composite solid figures. They apply volume formulas and use their previous experience with solving equations (**6.EE.B.7**) to find missing volumes and missing dimensions.

Module 5 concludes with deconstructing the faces of solid figures to determine surface area. Students note the difference between finding the volume of right rectangular prisms and finding the surface area of such prisms. In Lesson 15, students build solid figures using nets. They note which nets compose specific solid figures and also understand when nets cannot compose a solid figure. From this knowledge, students deconstruct solid figures into nets to identify the measurement of the solids' face edges. With this knowledge from Lesson 16, students are prepared to use nets to determine the surface area of solid figures in Lesson 17. They find that adding the areas of each face of the solid will result in a combined surface area. In Lesson 18, students find that each right rectangular prism has a front, a back, a top, a bottom, and two sides. They determine that surface area is obtained by adding the areas of all the faces. They understand that the front and back of the prism have the same surface area, the top and bottom have the same surface area, and the sides have the same surface area. Thus, students develop the formula $SA = 2lw + 2lh + 2wh$ (**6.G.A.4**). To wrap up the module, students apply the surface area formula to real-life contexts and distinguish between the need to find surface area or volume within contextual situations.

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Focus Standards

Solve real-world and mathematical problems involving area, surface area, and volume.

6.G.A.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

6.G.A.2 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

6.G.A.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

6.G.A.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Foundational Standards

Reason with shapes and their attributes.

1.G.A.2 Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.[2]

2.G.A.2 Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

3.G.A.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD.A.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*

[2] Students do not need to learn formal names such as "right rectangular prism."

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Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

4.G.A.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

5.NF.B.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.)*

5.NF.B.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[3]

Geometric measurement: understand conceptual concepts of volume and relate volume to multiplication and to addition.

5.MD.C.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.

b. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.

5.MD.C.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

5.MD.C.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

[3] Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

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c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

Graph points on a coordinate plane to solve real-world and mathematical problems.

5.G.A.1 Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).

5.G.A.2 Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Classify two-dimensional figures into categories based on their properties.

5.G.B.3 Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*

Apply and extend previous understandings of numbers to the system of rational numbers.

6.NS.C.8 Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Reason about and solve one-variable equations and inequalities.

6.EE.B.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

Focus Standards for Mathematical Practice

MP.1 **Make sense of problems and persevere in solving them.** Students make sense of real-world problems that involve area, volume, and surface area. One problem will involve multiple steps without breaking the problem into smaller, simpler questions. To solve surface area problems, students will have to find the area of different parts of the polygon before calculating the total area.

MP.3 **Construct viable arguments and critique the reasoning of others**. Students will develop different arguments as to why area formulas work for different polygons. Through this development, students may discuss and question their peers' thinking process. When students draw nets to represent right rectangular prisms, their representations may be

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different from their peers'. Although more than one answer may be correct, students will have an opportunity to defend their answers as well as question their peers. Students may also solve real-world problems using different methods; therefore, they may have to explain their thinking and critique their peers.

MP.4 **Model with mathematics.** Models will be used to demonstrate why the area formulas for different quadrilaterals are accurate. Students will use unit cubes to build right rectangular prisms and use these to calculate volume. The unit cubes will be used to model that $V = lwh$ and $V = bh$, where b represents the area of the base, are both accurate formulas to calculate the volume of a right rectangular prism. Students will use nets to model the process of calculating the surface area of a right rectangular prism.

MP.6 **Attend to precision.** Students will understand and use labels correctly throughout the module. For example, when calculating the area of a triangle, the answer will be labeled *units*^{2} because the area is the product of two dimensions. When two different units are given within a problem, students know to use previous knowledge of conversions to make the units match before solving the problem. In multi-step problems, students solve each part of the problem separately and know when to round in order to calculate the most precise answer. Students will attend to precision of language when describing exactly how a region may be composed or decomposed to determine its area.

Terminology

New or Recently Introduced Terms

- **Altitude and Base of a Triangle** (An *altitude* of a triangle is a perpendicular segment from a vertex of a triangle to the line containing the opposite side. The opposite side is called the *base*. For every triangle, there are three choices for the altitude, and hence there are three base-altitude pairs. The *height* of a triangle is the length of the altitude. The length of the base is called either the *base length* or, more commonly, the *base.* Usually, context makes it clear whether the *base* refers to a number or a segment. These terms can mislead students: base suggests the bottom, while *height* usually refers to vertical distances. Do not reinforce these impressions by consistently displaying all triangles with horizontal bases.)
- **Cube** (A *cube* is a right rectangular prism all of whose edges are of equal length.)
- **Hexagon** (Given 6 different points A, B, C, D, E, and F in the plane, a 6-*sided polygon,* or *hexagon,* is the union of 6 segments $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EF}$, and $\overline{FA}$ such that (1) the segments intersect only at their endpoints, and (2) no two adjacent segments are collinear. For both pentagons and hexagons, the segments are called the *sides*, and their endpoints are called the *vertices*. Like quadrilaterals, pentagons and hexagons can be denoted by the order of vertices defining the segments. For example, the pentagon $ABCDE$ has vertices A, B, C, D, and E that define the 5 segments in the definition above. Similar to quadrilaterals, pentagons and hexagons also have *interiors*, which can be described using pictures in elementary school.)
- **Line Perpendicular to a Plane** (A line L intersecting a plane E at a point P is said to be *perpendicular to the plane* E if L is perpendicular to every line that (1) lies in E and (2) passes through the point P. A segment is said to be perpendicular to a plane if the line that contains the segment is

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perpendicular to the plane. In Grade 6, a line perpendicular to a plane can be described using a picture.)

- **Parallel Planes** (Two planes are *parallel* if they do not intersect. In Euclidean geometry, a useful test for checking whether two planes are parallel is if the planes are different and if there is a line that is perpendicular to both planes.)
- **Pentagon** (Given 5 different points A, B, C, D, and E in the plane, a 5-*sided polygon,* or *pentagon*, is the union of 5 segments $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, and $\overline{EA}$ such that (1) the segments intersect only at their endpoints, and (2) no two adjacent segments are collinear.)
- **Right Rectangular Prism** (Let E and E' be two parallel planes. Let B be a rectangular region[4] in the plane E. At each point P of B, consider the segment PP' perpendicular to E, joining P to a point P' of the plane E'. The union of all these segments is called a *right rectangular prism.* It can be shown that the region B' in E' corresponding to the region B is also a rectangular region whose sides are equal in length to the corresponding sides of B. The regions B and B' are called the *base faces* (or just *bases*) of the prism. It can also be shown that the planar region between two corresponding sides of the bases is also a rectangular region called the *lateral face* of the prism. In all, the boundary of a right rectangular prism has 6 *faces*: the 2 base faces and 4 lateral faces. All adjacent faces intersect along segments called *edges*—base edges and lateral edges.)
- **Surface of a Prism** (The *surface of a prism* is the union of all of its faces—the base faces and lateral faces.)
- **Triangular Region** (A *triangular region* is the union of the triangle and its interior.)

Familiar Terms and Symbols[5]

- Angle
- Area
- Length of a Segment
- Parallel
- Parallelogram
- Perimeter
- Perpendicular
- Quadrilateral
- Rectangle
- Segment
- Square
- Trapezoid
- Triangle
- Volume

[4] A rectangular region is the union of a rectangle and its interior.
[5] These are terms and symbols students have seen previously.

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Suggested Tools and Representations

- Coordinate Planes
- Nets
- Prisms
- Rulers

Rapid White Board Exchanges

Implementing an RWBE requires that each student be provided with a personal white board, a white board marker, and a means of erasing his or her work. An economic choice for these materials is to place sheets of card stock inside sheet protectors to use as the personal white boards and to cut sheets of felt into small squares to use as erasers.

An RWBE consists of a sequence of 10 to 20 problems on a specific topic or skill that starts out with a relatively simple problem and progressively gets more difficult. The teacher should prepare the problems in a way that allows him or her to reveal them to the class one at a time. A flip chart or PowerPoint presentation can be used, or the teacher can write the problems on the board and either cover some with paper or simply write only one problem on the board at a time.

The teacher reveals, and possibly reads aloud, the first problem in the list and announces, "Go." Students work the problem on their personal white boards as quickly as possible and hold their work up for their teacher to see their answers as soon as they have the answer ready. The teacher gives immediate feedback to each student, pointing and/or making eye contact with the student and responding with an affirmation for correct work such as, "Good job!", "Yes!", or "Correct!", or responding with guidance for incorrect work such as "Look again," "Try again," "Check your work," etc. In the case of the RWBE, it is not recommended that the feedback include the name of the student receiving the feedback.

If many students have struggled to get the answer correct, go through the solution of that problem as a class before moving on to the next problem in the sequence. Fluency in the skill has been established when the class is able to go through each problem in quick succession without pausing to go through the solution of each problem individually. If only one or two students have not been able to successfully complete a problem, it is appropriate to move the class forward to the next problem without further delay; in this case find a time to provide remediation to that student before the next fluency exercise on this skill is given.

Sprints

Sprints are designed to develop fluency. They should be fun, adrenaline-rich activities that intentionally build energy and excitement. A fast pace is essential. During Sprint administration, teachers assume the role of athletic coaches. A rousing routine fuels students' motivation to do their personal best. Student recognition of increasing success is critical, and so every improvement is acknowledged. (See the Sprint Delivery Script for the suggested means of acknowledging and celebrating student success.)

One Sprint has two parts with closely-related problems on each. Students complete the two parts of the Sprint in quick succession with the goal of improving on the second part, even if only by one more.

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Sprints are not to be used for a grade. Thus, there is no need for students to write their names on the Sprints. The low-stakes nature of the exercise means that even students with allowances for extended time can participate. When a particular student finds the experience undesirable, it is recommended that the student be allowed to opt-out and take the Sprint home. In this case, it is ideal if the student has a regular opportunity to express the desire to opt-in.

With practice, the Sprint routine takes about 8 minutes.

Sprint Delivery Script

Gather the following: stopwatch, a copy of Sprint A for each student, a copy of Sprint B for each student, answers for Sprint A and Sprint B. The following delineates a script for delivery of a pair of Sprints.

This sprint covers: *topic.*

Do not look at the Sprint; keep it turned face down on your desk.

There are xx problems on the Sprint. You will have 60 seconds. Do as many as you can. I do not expect any of you to finish.

On your mark, get set, GO.

60 seconds of silence.

STOP. Circle the last problem you completed.

I will read the answers. You say "YES" if your answer matches. Mark the ones you have wrong. Don't try to correct them.

Energetically, rapid-fire call the answers ONLY.

Stop reading answers after there are no more students answering, "Yes."

Fantastic! Count the number you have correct, and write it on the top of the page. This is your personal goal for Sprint B.

Raise your hand if you have 1 or more correct. 2 or more, 3 or more...

Let us all applaud our runner-up, [insert name], with x correct. And let us applaud our winner, [insert name], with x correct.

You have a few minutes to finish up the page and get ready for the next Sprint.

Students are allowed to talk and ask for help; let this part last as long as most are working seriously.

Stop working. I will read the answers again so you can check your work. You say "YES" if your answer matches.

Energetically, rapid-fire call the answers ONLY.

Optionally, ask students to stand, and lead them in an energy-expanding exercise that also keeps the brain going. Examples are jumping jacks or arm circles, etc., while counting by 15's starting at 15, going up to 150 and back down to 0. You can follow this first exercise with a cool down exercise of a similar nature, such as calf raises with counting by one-sixths $\left(\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}, 1 \ldots\right)$.

Hand out the second Sprint, and continue reading the script.

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Keep the Sprint face down on your desk.

There are xx problems on the Sprint. You will have 60 seconds. Do as many as you can. I do not expect any of you to finish.

On your mark, get set, GO.

60 seconds of silence.

STOP. Circle the last problem you completed.

I will read the answers. You say "YES" if your answer matches. Mark the ones you have wrong. Don't try to correct them.

Quickly read the answers ONLY.

Count the number you have correct, and write it on the top of the page.

Raise your hand if you have 1 or more correct. 2 or more, 3 or more, ...

Let us all applaud our runner-up, [insert name], with x correct. And let us applaud our winner, [insert name], with x correct.

Write the amount by which your score improved at the top of the page.

Raise your hand if you improved your score by 1 or more. 2 or more, 3 or more, ...

Let us all applaud our runner-up for most improved, [insert name]. And let us applaud our winner for most improved, [insert name].

You can take the Sprint home and finish it if you want.

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	6.G.A.1, 6.G.A.3
End-of-Module Assessment Task	After Topic D	Constructed response with rubric	6.G.A.1, 6.G.A.2, 6.G.A.3, 6.G.A.4

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