



Table of Contents¹

Rational Numbers

Module Overview	3
Topic A: Addition and Subtraction of Integers and Rational Numbers (7.NS.A.1)	19
Lesson 1: Opposite Quantities Combine to Make Zero	21
Lesson 2: Using the Number Line to Model the Addition of Integers	31
Lesson 3: Understanding Addition of Integers	40
Lesson 4: Efficiently Adding Integers and Other Rational Numbers	49
Lesson 5: Understanding Subtraction of Integers and Other Rational Numbers	61
Lesson 6: The Distance Between Two Rational Numbers	71
Lesson 7: Addition and Subtraction of Rational Numbers	79
Lessons 8–9: Applying the Properties of Operations to Add and Subtract Rational Numbers	87
Topic B: Multiplication and Division of Integers and Rational Numbers (7.NS.A.2)	113
Lesson 10: Understanding Multiplication of Integers	115
Lesson 11: Develop Rules for Multiplying Signed Numbers	123
Lesson 12: Division of Integers	131
Lesson 13: Converting Between Fractions and Decimals Using Equivalent Fractions	140
Lesson 14: Converting Rational Numbers to Decimals Using Long Division	147
Lesson 15: Multiplication and Division of Rational Numbers	157
Lesson 16: Applying the Properties of Operations to Multiply and Divide Rational Numbers	168
Mid-Module Assessment and Rubric	182
<i>Topics A through B (assessment 1 day, return 1 day, remediation or further applications 1 day)</i>	
Topic C: Applying Operations with Rational Numbers to Expressions and Equations (7.NS.A.3, 7.EE.A.2, 7.EE.B.4a)	199
Lesson 17: Comparing Tape Diagram Solutions to Algebraic Solutions	201
Lessons 18–19: Writing, Evaluating, and Finding Equivalent Expressions with Rational Numbers	214
Lesson 20: Investments—Performing Operations with Rational Numbers	236

¹ Each lesson is ONE day, and ONE day is considered a 45-minute period.

Lesson 21: If-Then Moves with Integer Number Cards 247

Lessons 22–23: Solving Equations Using Algebra 257

End-of-Module Assessment and Rubric 278

Topics A through C (assessment 1 day, return 1 day, remediation or further applications 2 days)

Grade 7 • Module 2

Rational Numbers

OVERVIEW

In Grade 6, students formed a conceptual understanding of integers through the use of the number line, absolute value, and opposites and extended their understanding to include the ordering and comparing of rational numbers (**6.NS.C.5**, **6.NS.C.6**, **6.NS.C.7**). This module uses the Integer Game: a card game that creates a conceptual understanding of integer operations and serves as a powerful mental model students can rely on during the module. Students build on their understanding of rational numbers to add, subtract, multiply, and divide signed numbers. Previous work in computing the sums, differences, products, and quotients of fractions and decimals serves as a significant foundation as well.

In Topic A, students return to the number line to model the addition and subtraction of integers (**7.NS.A.1**). They use the number line and the Integer Game to demonstrate that an integer added to its opposite equals zero, representing the additive inverse (**7.NS.A.1a**, **7.NS.A.1b**). Their findings are formalized as students develop rules for adding and subtracting integers, and they recognize that subtracting a number is the same as adding its opposite (**7.NS.A.1c**). Real-life situations are represented by the sums and differences of signed numbers. Students extend integer rules to include the rational numbers and use properties of operations to perform rational number calculations without the use of a calculator (**7.NS.A.1d**).

Students develop the rules for multiplying and dividing signed numbers in Topic B. They use the properties of operations and their previous understanding of multiplication as repeated addition to represent the multiplication of a negative number as repeated subtraction (**7.NS.A.2a**). Students make analogies to the Integer Game to understand that the product of two negative numbers is a positive number. From earlier grades, they recognize division as the inverse process of multiplication. Thus, signed number rules for division are consistent with those for multiplication, provided a divisor is not zero (**7.NS.A.2b**). Students represent the division of two integers as a fraction, extending product and quotient rules to all rational numbers. They realize that any rational number in fractional form can be represented as a decimal that either terminates in 0s or repeats (**7.NS.A.2d**). Students recognize that the context of a situation often determines the most appropriate form of a rational number, and they use long division, place value, and equivalent fractions to fluently convert between these fractions and decimal forms. Topic B concludes with students multiplying and dividing rational numbers using the properties of operations (**7.NS.A.2c**).

In Topic C, students problem-solve with rational numbers and draw upon their work from Grade 6 with expressions and equations (**6.EE.A.2**, **6.EE.A.3**, **6.EE.A.4**, **6.EE.B.5**, **6.EE.B.6**, **6.EE.B.7**). They perform operations with rational numbers (**7.NS.A.3**), incorporating them into algebraic expressions and equations. They represent and evaluate expressions in multiple forms, demonstrating how quantities are related (**7.EE.A.2**). The Integer Game is revisited as students discover “if-then” statements, relating changes in player’s hands (who have the same card-value totals) to changes in both sides of a number sentence. Students translate word problems into algebraic equations and become proficient at solving equations of the form $px + q = r$ and $p(x + q) = r$, where p , q , and r , are specific rational numbers (**7.EE.B.4a**). As they

become fluent in generating algebraic solutions, students identify the operations, inverse operations, and order of steps, comparing these to an arithmetic solution. Use of algebra to represent contextual problems continues in Module 3.

This module is comprised of 23 lessons; 7 days are reserved for administering the Mid- and End-of-Module Assessments, returning the assessments, and remediating or providing further applications of the concepts. The Mid-Module Assessment follows Topic B, and the End-of-Module Assessment follows Topic C.

Focus Standards

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

- 7.NS.A.1** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
- Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*
 - Understand $p + q$ as the number located a distance $|q|$ from p , in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
 - Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
 - Apply properties of operations as strategies to add and subtract rational numbers.
- 7.NS.A.2** Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
- Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
 - Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.
 - Apply properties of operations as strategies to multiply and divide rational numbers.
 - Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

- 7.NS.A.3** Solve real-world and mathematical problems involving the four operations with rational numbers.²

Use properties of operations to generate equivalent expressions.

- 7.EE.A.2³** Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, $a + 0.05a = 1.05a$ means that “increase by 5%” is the same as “multiply by 1.05.”*

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

- 7.EE.B.4⁴** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
- a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p , q , and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

Foundational Standards

Use equivalent fractions as a strategy to add and subtract fractions.

- 5.NF.A.1** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, $2/3 + 5/4 = 8/12 + 15/12 = 23/12$. (In general, $a/b + c/d = (ad + bc)/bd$.)*

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

- 5.NF.B.3** Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret $3/4$ as the result of dividing 3 by 4, noting that $3/4$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $3/4$. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

² Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

³ In this module, this standard is applied to expressions with rational numbers in them.

⁴ In this module, the equations include negative rational numbers.

- 5.NF.B.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
- Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.)

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

- 6.NS.A.1** Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for $(2/3) \div (3/4)$ and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that $(2/3) \div (3/4) = 8/9$ because $3/4$ of $8/9$ is $2/3$. (In general, $(a/b) \div (c/d) = ad/bc$.) How much chocolate will each person get if 3 people share $1/2$ lb of chocolate equally? How many $3/4$ -cup servings are in $2/3$ of a cup of yogurt? How wide is a rectangular strip of land with length $3/4$ mi and area $1/2$ square mi?

Compute fluently with multi-digit numbers and find common factors and multiples.

- 6.NS.B.3** Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Apply and extend previous understandings of numbers to the system of rational numbers.

- 6.NS.C.5** Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
- 6.NS.C.6** Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
- Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite.
- 6.NS.C.7** Understand ordering and absolute value of rational numbers.
- Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of -30 dollars, write $|-30| = 30$ to describe the size of the debt in dollars.

Apply and extend previous understandings of arithmetic to algebraic expressions.

- 6.EE.A.2** Write, read, and evaluate expressions in which letters stand for numbers.
- Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation “Subtract y from 5” as $5 - y$.*
 - Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression $2(8 + 7)$ as a product of two factors; view $(8 + 7)$ as both a single entity and a sum of two terms.*
 - Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = 1/2$.*
- 6.EE.A.3** Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression $3(2 + x)$ to produce the equivalent expression $6 + 3x$; apply the distributive property to the expression $24x + 18y$ to produce the equivalent expression $6(4x + 3y)$; apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$.*
- 6.EE.A.4** Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions $y + y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for.*

Reason about and solve one-variable equations and inequalities.

- 6.EE.B.6** Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
- 6.EE.B.7** Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p , q and x are all nonnegative rational numbers.

Focus Standards for Mathematical Practice

- MP.1** **Make sense of problems and persevere in solving them.** When problem-solving, students use a variety of techniques to make sense of a situation involving rational numbers. For example, they may draw a number line and use arrows to model and make sense of an integer addition or subtraction problem. Or when converting between forms of rational numbers, students persevere in carrying out the long division algorithm to determine a decimal’s repeat pattern. A tape diagram may be constructed as an entry point to make sense of a working-backwards problem. As students fluently solve word problems using algebraic equations and

inverse operations, they consider their steps and determine whether or not they make sense in relationship to the arithmetic reasoning that served as their foundation in earlier grades.

- MP.2 Reason abstractly and quantitatively.** Students make sense of integer addition and subtraction through the use of an integer card game and diagramming the distances and directions on the number line. They use different properties of operations to add, subtract, multiply, and divide rational numbers, applying the properties to generate equivalent expressions or explain a rule. Students use integer subtraction and absolute value to justify the distance between two numbers on the number line. Algebraic expressions and equations are created to represent relationships. Students know how to use the properties of operations to solve equations. They make “zeros and ones” when solving an algebraic equation, thereby demonstrating an understanding of how the use of inverse operations ultimately leads to the value of the variable.
- MP.4 Model with mathematics.** Through the use of number lines, tape diagrams, expressions, and equations, students model relationships between rational numbers. Students relate operations involving integers to contextual examples. For instance, an overdraft fee of \$25 that is applied to an account balance of $-\$73.06$, is represented by the expression $-73.06 - 25$ or $-73.06 + (-25)$ using the additive inverse. Students compare their answers and thought processes in the Integer Game and use number line diagrams to ensure accurate reasoning. They deconstruct a difficult word problem by writing an equation, drawing a number line, or drawing a tape diagram to represent quantities. To find a change in elevation, students may draw a picture representing the objects and label their heights to aid in their understanding of the mathematical operation(s) that must be performed.
- MP.6 Attend to precision.** In performing operations with rational numbers, students understand that the decimal representation reflects the specific place value of each digit. When converting fractions to decimals, they carry out their calculations to specific place values, indicating a terminating or repeating pattern. In stating answers to problems involving signed numbers, students use integer rules and properties of operations to verify that the sign of their answer is correct. For instance, when finding an average temperature for temperatures whose sum is a negative number, students realize that the quotient must be a negative number since the divisor is positive and the dividend is negative.
- MP.7 Look for and make use of structure.** Students formulate rules for operations with signed numbers by observing patterns. For instance, they notice that adding -7 to a number is the same as subtracting 7 from the number, and thus, they develop a rule for subtraction that relates to adding the inverse of the subtrahend. Students use the concept of absolute value and subtraction to represent the distance between two rational numbers on a number line. They use patterns related to the properties of operations to justify the rules for multiplying and dividing signed numbers. The order of operations provides the structure by which students evaluate and generate equivalent expressions.

Terminology

New or Recently Introduced Terms

- **Additive Identity** (The additive identity is 0.)
- **Additive Inverse** (The *additive inverse* of a real number is the opposite of that number on the real number line. For example, the opposite of -3 is 3. A number and its additive inverse have a sum of 0.)
- **Break-Even Point** (The *break-even point* is the point at which there is neither a profit nor loss.)
- **Distance Formula** (If p and q are rational numbers on a number line, then the distance between p and q is $|p - q|$.)
- **Loss** (A decrease in amount, as when the money earned is less than the money spent.)
- **Multiplicative Identity** (The *multiplicative identity* is 1.)
- **Profit** (A gain, as in the positive amount represented by the difference between the money earned and spent)
- **Repeating Decimal** (The decimal form of a rational number, for example, $\frac{1}{3} = 0.\overline{3}$.)
- **Terminating Decimal** (A decimal is called terminating if its repeating digit is 0.)

Familiar Terms and Symbols⁵

- Absolute Value
- Associative Property (of Multiplication and Addition)
- Commutative Property (of Multiplication and Addition)
- Credit
- Debit
- Deposit
- Distributive Property (of Multiplication Over Addition)
- Expression
- Equation
- Integer
- Inverse
- Multiplicative Inverse
- Opposites
- Overdraft
- Positives
- Negatives

⁵ These are terms and symbols students have seen previously.

- Rational Numbers
- Withdraw

Suggested Tools and Representations

- Equations
- Expressions
- Integer Game (See explanation on page 11)
- Number Line
- Tape Diagram

Sprints

Sprints are designed to develop fluency. They should be fun, adrenaline-rich activities that intentionally build energy and excitement. A fast pace is essential. During Sprint administration, teachers assume the role of athletic coaches. A rousing routine fuels students' motivation to do their personal best. Student recognition of increasing success is critical, and so every improvement is acknowledged. (See the Sprint Delivery Script for the suggested means of acknowledging and celebrating student success.)

One Sprint has two parts with closely-related problems on each. Students complete the two parts of the Sprint in quick succession with the goal of improving on the second part, even if only by one more.

Sprints are not to be used for a grade. Thus, there is no need for students to write their names on the Sprints. The low-stakes nature of the exercise means that even students with allowances for extended time can participate. If a student finds the experience undesirable, it is recommended that the student be allowed to opt-out and take the Sprint home. In this case, regularly encourage the student to opt back in to participate in class.

With practice, the Sprint routine takes about 8 minutes.

Sprint Delivery Script

Gather the following: stopwatch, a copy of Sprint A for each student, a copy of Sprint B for each student, answers for Sprint A and Sprint B. The following delineates a script for delivery of a pair of Sprints.

This sprint covers: *topic*.

Do not look at the Sprint, keep it turned face down on your desk.

There are xx problems on the Sprint. You will have 60 seconds. Do as many as you can. I do not expect any of you to finish.

On your mark, get set, GO.

60 seconds of silence.

STOP. Circle the last problem you completed.

I will read the answers. You say "YES" if your answer matches. Mark the ones you have wrong. Don't try

to correct them.

Energetically, rapid-fire call the answers ONLY.

Stop reading answers after there are no more students answering, “Yes.”

Fantastic! Count the number you have correct, and write it on the top of the page. This is your personal goal for Sprint B.

Raise your hand if you have 1 or more correct. 2 or more, 3 or more...

Let us all applaud our runner up, [insert name] with x correct. And let us applaud our winner, [insert name], with x correct.

You have a few minutes to finish up the page and get ready for the next Sprint.

Students are allowed to talk and ask for help; let this part last as long as most are working seriously.

Stop working. I will read the answers again so you can check your work. You say “YES” if your answer matches.

Energetically, rapid-fire call the answers ONLY.

Optionally, ask students to stand and lead them in an energy-expanding exercise that also keeps the brain going. Examples are jumping jacks or arm circles, etc. while counting by 15’s starting at 15, going up to 150 and back down to 0. You can follow this first exercise with a cool down exercise of a similar nature, such as calf raises with counting by one-sixths $(\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}, 1 \dots)$.

Hand out the second Sprint and continue reading the script.

Keep the Sprint face down on your desk.

There are xx problems on the Sprint. You will have 60 seconds. Do as many as you can. I do not expect any of you to finish.

On your mark, get set, GO.

60 seconds of silence.

STOP. Circle the last problem you completed.

I will read the answers. You say “YES” if your answer matches. Mark the ones you have wrong. Don’t try to correct them.

Quickly read the answers ONLY.

Count the number you have correct, and write it on the top of the page.

Raise your hand if you have 1 or more correct. 2 or more, 3 or more, ...

Let us all applaud our runner up, [insert name] with x correct. And let us applaud our winner, [insert name], with x correct.

Write the amount by which your score improved at the top of the page.

Raise your hand if you improved your score by 1 or more. 2 or more, 3 or more...

Let us all applaud our runner up for most improved, [insert name]. And let us applaud our winner for most improved, [insert name].

You can take the Sprint home and finish it if you want.

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	7.NS.A.1, 7.NS.A.2
End-of-Module Assessment Task	After Topic C	Constructed response with rubric	7.NS.A.1, 7.NS.A.2, 7.NS.A.3, 7.EE.A.2, 7.EE.B.4a