Table of Contents[1]

Percent and Proportional Relationships

[1] Each lesson is ONE day, and ONE day is considered a 45-minute period.

Grade 7 • Module 4

Percent and Proportional Relationships

OVERVIEW

In Module 4, students deepen their understanding of ratios and proportional relationships from Module 1 (**7.RP.A.1**, **7.RP.A.2**, **7.RP.A.3**, **7.EE.B.4**, **7.G.A.1**) by solving a variety of percent problems. They convert between fractions, decimals, and percents to further develop a conceptual understanding of percent (introduced in Grade 6, Module 1) and use algebraic expressions and equations to represent and solve multi-step percent scenarios (**7.EE.B.3**). An initial focus on relating 100% to "the whole" serves as a foundation for students. Students begin the module by solving problems without the use of a calculator to develop a greater fluency and deeper reasoning behind calculations with percent. Material in early lessons is designed to reinforce students' understanding by having them use mental math and basic computational skills. To develop a conceptual understanding, students will use visual models and equations, building on earlier work with these strategies. As the lessons and topics progress and more complex calculations are required to solve multi-step percent problems, teachers may let students use calculators so that their computational fluency does not interfere with the primary concept(s) being addressed. This will also be noted in the teacher's lesson materials.

Topic A builds on students' conceptual understanding of percent from Grade 6 (**6.RP.A.3c**) and relates 100% to "the whole." Students represent percents as decimals and fractions and extend their understanding from Grade 6 to include percents greater than 100%, such as 225%, and percents less than 1%, such as $\frac{1}{2}\%$ or 0.5%. They understand that, for instance, 225% means $\frac{225}{100}$, which ultimately simplifies to the equivalent decimal value of 2.25 (**7.RP.A.1**). Students use complex fractions to represent non-whole number percents (e.g., $12\frac{1}{2}\% = \frac{12\frac{1}{2}}{100} = \frac{1}{8} = 0.125$).

Module 3's focus on algebra prepares students to move from the visual models used for percents in Grade 6 to algebraic equations in Grade 7. They write equations to solve multi-step percent problems and relate their conceptual understanding to the representation: $\text{Quantity} = \text{Percent} \times \text{Whole}$ (**7.RP.A.2c**). Students solve percent increase and decrease problems with and without equations (**7.RP.A.3**). For instance, given a multi-step word problem where there is an increase of 20% and "the whole" equals $\$200$, students recognize that $\$200$ can be multiplied by 120%, or 1.2, to get an answer of $\$240$. They use visual models such as a double number line diagram to justify their answers. In this case, 100% aligns to $\$200$ in the diagram, and intervals of fifths are used (since $20\% = \frac{1}{5}$) to partition both number line segments to create a scale indicating that 120% aligns to $\$240$. Topic A concludes with students representing 1% of a quantity using a ratio and then using that ratio to find the amounts of other percents. While representing 1% of a quantity and using it to find the amount of other percents is a strategy that will always work when solving a problem, students recognize that when the percent is a factor of 100, they can use mental math and proportional reasoning to find the amount of other percents in a more efficient way.

In Topic B, students create algebraic representations and apply their understanding of percent from Topic A to interpret and solve multi-step word problems related to markups or markdowns, simple interest, sales tax, commissions, fees, and percent error (**7.RP.A.3**, **7.EE.B.3**). They apply their understanding of proportional relationships from Module 1, creating an equation, graph, or table to model a tax or commission rate that is represented as a percent (**7.RP.A.1**, **7.RP.A.2**). Students solve problems related to changing percents and use their understanding of percent and proportional relationships to solve scenarios such as the following: A soccer league has 300 players, 60% of whom are boys. If some of the boys switch to baseball, leaving only 52% of the soccer players as boys, how many players remain in the soccer league? Students first determine that $100\% - 60\% = 40\%$ of the players are girls, and 40% of 300 equals 120. Then, after some boys switched to baseball, $100\% - 52\% = 48\%$ of the soccer players are girls; so, $0.48p = 120$, or $p = \frac{120}{0.48}$. Therefore, there are now 250 players in the soccer league.

In Topic B, students also apply their understanding of absolute value from Module 2 (**7.NS.A.1b**) when solving percent error problems. To determine the percent error for an estimated concert attendance of $5{,}000$ people, when actually $6{,}372$ people attended, students calculate the percent error as $\frac{|5000-6372|}{|6372|} \cdot 100\%$, which is about 21.5%.

Students revisit scale drawings in Topic C to solve problems in which the scale factor is represented by a percent (**7.RP.A.2b**, **7.G.A.1**). They understand from their work in Module 1, for example, that if they have two drawings, and if Drawing 2 is a scale model of Drawing 1 under a scale factor of 80%, then Drawing 1 is also a scale model of Drawing 2, and that scale factor is determined using inverse operations. Since $80\% = \frac{4}{5}$, the scale factor is found by taking the complex fraction $\frac{1}{\frac{4}{5}}$, or $\frac{5}{4}$, and multiplying it by 100%, resulting in a scale factor of 125%. As in Module 1, students construct scale drawings, finding scale lengths and areas given the actual quantities and the scale factor (and vice versa); however, in this module the scale factor is represented as a percent. Students are encouraged to develop multiple methods for making scale drawings. Students may find the multiplicative relationship between figures; they may also find a multiplicative relationship among lengths within the same figure.

The problem-solving materials in Topic D provide students with further applications of percent and exposure to problems involving population, mixtures, and counting in preparation for later topics in middle school and high school mathematics and science. Students will apply their understanding of percent (**7.RP.A.2c**, **7.RP.A.3**, **7.EE.B.3**) to solve complex word problems by identifying a set that meets a certain percentage criterion. Additionally, students will explore problems involving mixtures of ingredients and determine the percentage of a mixture that already exists, or on the contrary, the amount of ingredient needed in order to meet a certain percentage criterion.

This module spans 25 days and includes 18 lessons. Seven days are reserved for administering the assessments, returning the assessments, and remediating or providing further applications of the concepts. The Mid-Module Assessment follows Topic B, and the End-of-Module Assessment follows Topic D.

Focus Standards

Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the complex fraction $\frac{1}{2}/\frac{1}{4}$ miles per hour, equivalently 2 miles per hour.*

7.RP.A.2 Recognize and represent proportional relationships between quantities.

a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.*

d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0,0)$ and $(1, r)$, where r is the unit rate.

7.RP.A.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.[2]

7.EE.B.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $\$25$ an hour gets a 10% raise, she will make an additional $\frac{1}{10}$ of her salary an hour, or $\$2.50$, for a new salary of $\$27.50$. If you want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door that is $27\frac{1}{2}$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

[2] 7.EE.B.3 is introduced in Module 3. The majority of this cluster was taught in the first three modules.

Draw, construct, and describe geometrical figures and describe the relationships between them.

7.G.A.1[3] Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Foundational Standards

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.A.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was* $2:1$*, because for every* 2 *wings there was* 1 *beak." "For every vote candidate A received, candidate C received nearly three votes."*

6.RP.A.2 Understand the concept of a unit rate $\frac{a}{b}$ associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of* 3 *cups of flour to* 4 *cups of sugar, so there is* $\frac{3}{4}$ *cup of flour for each cup of sugar." "We paid* $\$75$ *for* 15 *hamburgers, which is a rate of* $\$5$ *per hamburger."*[4]

6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took* 7 *hours to mow* 4 *lawns, then at that rate, how many lawns could be mowed in* 35 *hours? At what rate were lawns being mowed?*

c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means $\frac{30}{100}$ times the quantity); solve problems involving finding the whole, given a part and the percent.

d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Solve real-world and mathematical problems involving area, surface area, and volume.

6.G.A.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

[3] 7.G.A.1 is introduced in Module 1. The balance of this cluster is taught in Module 6.
[4] Expectations for unit rates in this grade are limited to non-complex fractions.

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

7.NS.A.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

7.NS.A.3 Solve real-world and mathematical problems involving the four operations with rational numbers.[5]

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.B.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is* 54 cm. *Its length is* 6 cm. *What is its width?*

Focus Standards for Mathematical Practice

MP.1 **Make sense of problems and persevere in solving them.** Students make sense of percent problems by modeling the proportional relationship using an equation, a table, a graph, a double number line diagram, mental math, and factors of 100. When solving a multi-step percent word problem, students use estimation and number sense to determine if their steps and logic lead to a reasonable answer. Students know they can always find 1% of a quantity by dividing it by 100 or multiplying it by $\frac{1}{100}$, and they also know that finding 1% first allows them to then find other percents easily. For instance, if students are trying to find the amount of money after 4 years in a savings account with an annual interest rate of $\frac{1}{2}\%$ on an account balance of $\$300$, they use the fact that 1% of 300 equals $\frac{300}{100}$, or $\$3$; thus, $\frac{1}{2}\%$ of 300 equals $\frac{1}{2}$ of $\$3$, or $\$1.50$. $\$1.50$ multiplied by 4 is $\$6$ interest, and adding $\$6$ to $\$300$ makes the total balance, including interest, equal to $\$306$.

[5] Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

MP.2 **Reason abstractly and quantitatively.** Students use proportional reasoning to recognize that when they find a certain percent of a given quantity, the answer must be greater than the given quantity if they found more than 100% of it and less than the given quantity if they found less than 100% of it. Double number line models are used to visually represent proportional reasoning related to percents in problems such as the following: If a father has 70% more money in his savings account than his 25-year-old daughter has in her savings account, and the daughter has $\$4{,}500$, how much is in the father's account? Students represent this information with a visual model by equating $4{,}500$ to 100% and the father's unknown savings amount to 170% of $4{,}500$. Students represent the amount of money in the father's savings account by writing the expression $\frac{170}{100} \times 4{,}500$, or $1.7(4{,}500)$. When working with scale drawings, given an original two-dimensional picture and a scale factor as a percent, students generate a scale drawing so that each corresponding measurement increases or decreases by a certain percentage of measurements of the original figure. Students work backward to create a new scale factor and scale drawing when given a scale factor represented as a percent greater or less than 100%. For instance, given a scale drawing with a scale factor of 25%, students create a new scale drawing with a scale factor of 10%. They relate working backward in their visual model to the following steps: (1) multiplying all lengths in the original scale drawing by $\frac{1}{0.25}$ (or dividing by 25%) to get back to their original lengths, and then (2) multiplying each original length by 10% to get the new scale drawing.

MP.5 **Use appropriate tools strategically.** Students solve word problems involving percents using a variety of tools, including equations and double number line models. They choose their model strategically. For instance, given that 75% of a class of learners is represented by 21 students, they recognize that since 75 is $\frac{3}{4}$ of 100, and 75 and 21 are both divisible by 3, a double number line diagram can be used to establish intervals of 25's and 7's to show that 100% would correspond to $21 + 7$, which equals 28. For percent problems that do not involve benchmark fractions, decimals, or percents, students use math sense and estimation to assess the reasonableness of their answers and computational work. For instance, if a problem indicates that a bicycle is marked up 18% and it is sold at a retail price of $\$599$, students are able to estimate by using rounded values such as 120% and $\$600$ to determine that the solution that will represent the wholesale price of the bicycle must be in the realm of $600 \div 1.2$, or $6{,}000 \div 12$, to arrive at an estimate of $\$500$.

MP.6 **Attend to precision.** Students pay close attention to the context of the situation when working with percent problems involving a percent markup, markdown, increase, or decrease. They construct models based on the language of a word problem. For instance, a markdown of 15% on an $\$88$ item is represented by $0.85(88)$; however, a markup of 15% is represented by $1.15(88)$. Students attend to precision when writing the answer to a percent problem. If they are finding a percent, they use the $\%$ symbol in the answer or write the answer as a fraction with 100 as the denominator (or in an equivalent form). Double number line diagrams display correct segment lengths, and if a line in the diagram represents percents, it is either labeled as such or the percent sign is shown after each number. When stating the area of a scale drawing or actual drawing, students include the square units along with the numerical part of the answer.

MP.7 **Look for and make use of structure**. Students understand percent to be a rate per 100 and express p percent as $\frac{p}{100}$. They know that, for instance, 5% means 5 for every 100, 1% means 1 for every 100, and 225% means 225 for every 100. They use their number sense to find benchmark percents. Since 100% is one whole, then 25% is one-fourth, 50% is one-half, and 75% is three-fourths. So, to find 75% of 24, they find $\frac{1}{4}$ of 24, which is 6, and multiply it by 3 to arrive at 18. They use factors of 100 and mental math to solve problems involving other benchmark percents as well. Students know that 1% of a quantity represents $\frac{1}{100}$ of it and use place value and the structure of the base-ten number system to find 1% or $\frac{1}{100}$ of a quantity. They use "finding 1%" as a method to solve percent problems. For instance, to find 14% of 245, students first find 1% of 245 by dividing 245 by 100, which equals 2.45. Since 1% of 245 equals 2.45, 14% of 245 would equal $2.45 \times 14 = 34.3$. Students observe the steps involved in finding a discount price or price including sales tax and use the properties of operations to efficiently find the answer. To find the discounted price of a $\$73$ item that is on sale for 15% off, students realize that the distributive property allows them to arrive at an answer in one step, by multiplying $\$73$ by 0.85, since $73(100\%) - 73(15\%) = 73(1) - 73(0.15) = 73(0.85)$.

Terminology

New or Recently Introduced Terms

- **Absolute Error** (Given the exact value x of a quantity and an approximate value a of it, the absolute error is $|a - x|$.)
- **Percent Error** (The percent error is the percent the absolute error is of the exact value $\left(\frac{|a-x|}{|x|}\right)(100\%)$, where x is the exact value of the quantity, and a is an approximate value of the quantity.)

Familiar Terms and Symbols[6]

- Area
- Circumference
- Coefficient of the Term
- Complex Fraction
- Constant of Proportionality
- Discount Price
- Equation
- Equivalent Ratios

[6] These are terms and symbols students have seen previously.

- Expression
- Fee
- Fraction
- Greatest Common Factor
- Length of a Segment
- One-to-One Correspondence
- Original Price
- Percent
- Perimeter
- Pi
- Proportional Relationship
- Proportional To
- Rate
- Ratio
- Rational Number
- Sales Price
- Scale Drawing
- Scale Factor
- Unit Rate

Suggested Tools and Representations

- Calculator
- Coordinate Plane
- Double Number Line Diagrams
- Equations
- Expressions
- Geometric Figures
- Ratio Tables
- Tape Diagrams

Sprints

Sprints are designed to develop fluency. They should be fun, adrenaline-rich activities that intentionally build energy and excitement. A fast pace is essential. During Sprint administration, teachers assume the role of athletic coaches. A rousing routine fuels students' motivation to do their personal best. Student recognition of increasing success is critical, and so every improvement is acknowledged. (See the Sprint Delivery Script for the suggested means of acknowledging and celebrating student success.)

One Sprint has two parts with closely related problems on each. Students complete the two parts of the Sprint in quick succession with the goal of improving on the second part, even if only by one more.

Sprints are not to be used for a grade. Thus, there is no need for students to write their names on the Sprints. The low-stakes nature of the exercise means that even students with allowances for extended time can participate. When a particular student finds the experience undesirable, it is recommended that the student be allowed to opt out and take the Sprint home. In this case, it is ideal if the student has a regular opportunity to express the desire to opt in.

With practice, the Sprint routine takes about 8 minutes.

Sprint Delivery Script

Gather the following: stopwatch, a copy of Sprint A for each student, a copy of Sprint B for each student, answers for Sprint A and Sprint B. The following delineates a script for delivery of a pair of Sprints.

This sprint covers: *topic.*

Do not look at the Sprint; keep it turned face down on your desk.

There are xx problems on the Sprint. You will have 60 seconds. Do as many as you can. I do not expect any of you to finish.

On your mark, get set, GO.

60 seconds of silence.

STOP. Circle the last problem you completed.

I will read the answers. You say "YES" if your answer matches. Mark the ones you have wrong. Don't try to correct them.

Energetically, rapid-fire call the answers ONLY.

Stop reading answers after there are no more students answering, "Yes."

Fantastic! Count the number you have correct, and write it on the top of the page. This is your personal goal for Sprint B.

Raise your hand if you have 1 or more correct. 2 or more, 3 or more...

Let us all applaud our runner-up, [insert name], with x correct. And let us applaud our winner, [insert name], with x correct.

You have a few minutes to finish up the page and get ready for the next Sprint.

Students are allowed to talk and ask for help; let this part last as long as most are working seriously.

Stop working. I will read the answers again so you can check your work. You say "YES" if your answer matches.

Energetically, rapid-fire call the answers ONLY.

Optionally, ask students to stand, and lead them in an energy-expanding exercise that also keeps the brain going. Examples are jumping jacks or arm circles, etc., while counting by 15's starting at 15, going up to 150 and back down to 0. You can follow this first exercise with a cool down exercise of a similar nature, such as calf raises with counting by one-sixths $\left(\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}, 1 \ldots\right)$.

Hand out the second Sprint, and continue reading the script.

Keep the Sprint face down on your desk.

There are xx problems on the Sprint. You will have 60 seconds. Do as many as you can. I do not expect any of you to finish.

On your mark, get set, GO.

60 seconds of silence.

STOP. Circle the last problem you completed.

I will read the answers. You say "YES" if your answer matches. Mark the ones you have wrong. Don't try to correct them.

Quickly read the answers ONLY.

Count the number you have correct, and write it on the top of the page.

Raise your hand if you have 1 or more correct. 2 or more, 3 or more, ...

Let us all applaud our runner-up, [insert name], with x correct. And let us applaud our winner, [insert name], with x correct.

Write the amount by which your score improved at the top of the page.

Raise your hand if you improved your score by 1 or more. 2 or more, 3 or more, ...

Let us all applaud our runner-up for most improved, [insert name]. And let us applaud our winner for most improved, [insert name].

You can take the Sprint home and finish it if you want.

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	7.RP.A.1, 7.RP.A.2, 7.RP.A.3, 7.EE.B.3
End-of-Module Assessment Task	After Topic D	Constructed response with rubric	7.RP.A.1, 7.RP.A.2, 7.RP.A.3, 7.EE.B.3, 7.G.A.1