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Grade 7 • Module 5

Statistics and Probability

OVERVIEW

In this module, students begin their study of probability, learning how to interpret probabilities and how to compute probabilities in simple settings. They also learn how to estimate probabilities empirically. The concept of probability provides a foundation for the thinking required to make inferential reasoning that will be developed in the second half of this module. Additionally, students build on their knowledge of data distributions that they studied in Grade 6, compare data distributions of two or more populations, and are introduced to the idea of drawing informal inferences based on data collected from random samples.

In Topics A and B, students learn to interpret the probability of an event as the proportion of the time that the event will occur when a chance experiment is repeated many times (**7.SP.C.5**). They learn to compute or estimate probabilities using a variety of methods, including collecting data, using tree diagrams, and using simulations. In Topic B, students move to comparing probabilities from simulations to computed probabilities that are based on theoretical models (**7.SP.C.6**, **7.SP.C.7**). They calculate probabilities of compound events using lists, tables, tree diagrams, and simulations (**7.SP.C.8**). They learn to use probabilities to make decisions and to determine whether or not a given probability model is plausible (**7.SP.C.7**). The Mid-Module Assessment follows Topic B.

In Topics C and D, students focus on using random sampling to draw informal inferences about a population (**7.SP.A.1**, **7.SP.A.2**). In Topic C, they investigate sampling from a population (**7.SP.A.2**). They learn to estimate a population mean using numerical data from a random sample (**7.SP.A.2**). They also learn how to estimate a population proportion using categorical data from a random sample. In Topic D, students learn to compare two populations with similar variability. They learn to consider sampling variability when deciding if there is evidence that the means or the proportions of two populations are actually different (**7.SP.B.3**, **7.SP.B.4**). The End-of-Module Assessment follows Topic D.

Focus Standards

Using random sampling to draw inferences about a population.

- 7.SP.A.1** Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

- 7.SP.A.2** Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*

Draw informal comparative inferences about two populations.

- 7.SP.B.3** Informally assess the degree of visual overlap of two numerical data distributions with similar variability, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*
- 7.SP.B.4** Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*

Investigate chance processes and develop, use, and evaluate probability models.

- 7.SP.C.5** Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $1/2$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
- 7.SP.C.6** Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*
- 7.SP.C.7** Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.
- Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*
 - Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*

- 7.SP.C.8** Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.
- Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
 - Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.
 - Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

Foundational Standards

Summarize and describe distributions.

- 6.SP.B.5** Summarize numerical data sets in relation to their context, such as by:
- Reporting the number of observations.
 - Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
 - Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
 - Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Understand ratio concepts and use ratio reasoning to solve problems.

- 6.RP.A.3c** Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

Analyze proportional relationships and use them to solve real-world and mathematical problems.

- 7.RP.A.2** Recognize and represent proportional relationships between quantities.

Focus Standards for Mathematical Practice

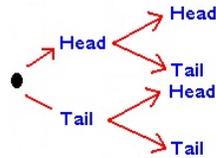
- MP.2 Reason abstractly and quantitatively.** Students reason quantitatively by posing statistical questions about variables and the relationship between variables. Students reason abstractly about chance experiments by analyzing possible outcomes and designing simulations to estimate probabilities.
- MP.3 Construct viable arguments and critique the reasoning of others.** Students construct viable arguments by using sample data to explore conjectures about a population. Students critique the reasoning of other students as part of poster or similar presentations.
- MP.4 Model with mathematics.** Students use probability models to describe outcomes of chance experiments. They evaluate probability models by calculating the theoretical probabilities of chance events, and by comparing these probabilities to observed relative frequencies.
- MP.5 Use appropriate tools strategically.** Students use simulation to approximate probabilities. Students use appropriate technology to calculate measures of center and variability. Students use graphical displays to visually represent distributions.
- MP.6 Attend to precision.** Students interpret and communicate conclusions in context based on graphical and numerical data summaries. Students make appropriate use of statistical terminology.

Terminology

New or Recently Introduced Terms

- **Compound Event** (*A compound event* is an event consisting of more than one outcome from the sample space of a chance experiment.)
- **Inference** (*Inference* is the act of drawing conclusions about a population using data from a sample.)
- **Long-Run Relative Frequency** (The proportion of the time some outcome occurs in a very long sequence of observations is called a *long-run relative frequency*.)
- **Probability** (Probability is a number between 0 and 1 that represents the likelihood that an outcome will occur.)
- **Probability Model** (*A probability model* for a chance experiment specifies the set of possible outcomes of the experiment—the sample space—and the probability associated with each outcome.)
- **Random Sample** (*A random sample* is a sample selected in a way that gives every different possible sample of the same size an equal chance of being selected.)
- **Simulation** (*A simulation* is the process of generating “artificial” data that are consistent with a given probability model or with sampling from a known population.)

- **Tree Diagram** (A *tree diagram* consists of a sequence of nodes and branches. Tree diagrams are sometimes used as a way of representing the outcomes of a chance experiment that consists of a sequence of steps, such as rolling two number cubes, viewed as first rolling one number cube and then rolling the second.)



- **Uniform Probability Model** (A *uniform probability model* is a probability model in which all outcomes in the sample space of a chance experiment are equally likely.)

Familiar Terms and Symbols²

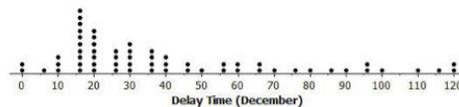
- Mean Absolute Deviation (MAD)
- Measures of Center
- Measures of Variability
- Shape

Suggested Tools and Representations

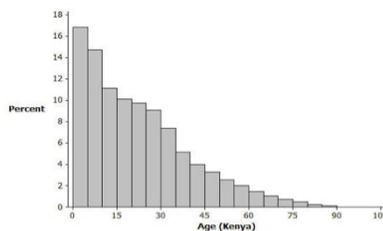
- Graphing Calculator (See example below.)
- Dot Plots (See example below.)
- Histograms (See example below.)



Graphing Calculator



Dot Plot



Histogram

² These are terms and symbols students have seen previously.

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	7.SP.C.5, 7.SP.C.6, 7.SP.C.7, 7.SP.C.8
End-of-Module Assessment Task	After Topic D	Constructed response with rubric	7.SP.A.1, 7.SP.A.2, 7.SP.B.3, 7.SP.B.4, 7.SP.C.5, 7.SP.C.6, 7.SP.C.7, 7.SP.C.8