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Integer Exponents and Scientific Notation

[1] Each lesson is ONE day, and ONE day is considered a 45-minute period.

Grade 8 • Module 1

Integer Exponents and Scientific Notation

OVERVIEW

In Module 1, students' knowledge of operations on numbers will be expanded to include operations on numbers in integer exponents. Module 1 also builds on students' understanding from previous grades with regard to transforming expressions. Students were introduced to exponential notation in Grade 5 as they used whole number exponents to denote powers of ten (**5.NBT.A.2**). In Grade 6, students expanded the use of exponents to include bases other than ten as they wrote and evaluated exponential expressions limited to whole-number exponents (**6.EE.A1**). Students made use of exponents again in Grade 7 as they learned formulas for the area of a circle (**7.G.B.4**) and volume (**7.G.B.6**).

In this module, students build upon their foundation with exponents as they make conjectures about how zero and negative exponents of a number should be defined and prove the properties of integer exponents (**8.EE.A.1**). These properties are codified into three laws of exponents. They make sense out of very large and very small numbers, using the number line model to guide their understanding of the relationship of those numbers to each other (**8.EE.A.3**).

Having established the properties of integer exponents, students learn to express the magnitude of a positive number through the use of scientific notation and to compare the relative size of two numbers written in scientific notation (**8.EE.A.3**). Students explore use of scientific notation and choose appropriately sized units as they represent, compare, and make calculations with very large quantities, such as the U.S. national debt, the number of stars in the universe, and the mass of planets; and very small quantities, such as the mass of subatomic particles (**8.EE.A.4**).

The Mid-Module Assessment follows Topic A. The End-of-Module Assessment follows Topic B.

Focus Standards

Work with radicals and integer exponents.

8.EE.A.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

8.EE.A.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9*, and determine that the world population is more than 20 times larger.*

8.EE.A.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Foundational Standards

Understand the place value system.

5.NBT.A.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

Apply and extend previous understandings of arithmetic to algebraic expressions.

6.EE.A.1 Write and evaluate numerical expressions involving whole-number exponents.

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

7.G.B.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

7.G.B.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Focus Standards for Mathematical Practice

MP.2 **Reason abstractly and quantitatively.** Students use concrete numbers to explore the properties of numbers in exponential form and then prove that the properties are true for all positive bases and all integer exponents using symbolic representations for bases and exponents. As lessons progress, students use symbols to represent integer exponents and make sense of those quantities in problem situations. Students refer to symbolic notation in order to contextualize the requirements and limitations of given statements (e.g., letting m, n represent positive integers, letting a, b represent all integers, both with respect to the properties of exponents).

MP.3 **Construct viable arguments and critique the reasoning of others.** Students reason through the acceptability of definitions and proofs (e.g., the definitions of x^0 and x^{-b} for all integers b and positive integers x). New definitions, as well as proofs, require students to analyze situations and break them into cases. Further, students examine the implications of these definitions and proofs on existing properties of integer exponents. Students keep the goal of a logical argument in mind while attending to details that develop during the reasoning process.

MP.6 **Attend to precision.** Beginning with the first lesson on exponential notation, students are required to attend to the definitions provided throughout the lessons and the limitations of symbolic statements, making sure to express what they mean clearly. Students are provided a hypothesis, such as $x < y$, for positive integers x, y, and then asked to evaluate whether a statement, like $-2 < 5$, contradicts this hypothesis.

MP.7 **Look for and make use of structure.** Students understand and make analogies to the distributive law as they develop properties of exponents. Students will know $x^m \cdot x^n = x^{m+n}$ as an analog of $mx + nx = (m+n)x$ and $(x^m)^n = x^{m \times n}$ as an analog of $n \times (m \times x) = (n \times m) \times x$.

MP.8 **Look for and express regularity in repeated reasoning.** While evaluating the cases developed for the proofs of laws of exponents, students identify when a statement must be proved or if it has already been proven. Students see the use of the laws of exponents in application problems and notice the patterns that are developed in problems.

Terminology

New or Recently Introduced Terms

- **Scientific Notation** (The scientific notation *for a finite decimal* is the representation of that decimal as the product of a decimal s and a power of 10, where s satisfies the property that it is at least 1, but smaller than 10, or in symbolic notation, $1 \leq s < 10$. For example, the scientific notation for 192.7 is 1.927×10^2.)
- **Order of Magnitude** (The order of magnitude *of a finite decimal* is the exponent in the power of 10 when that decimal is expressed in scientific notation. For example, the order of magnitude of 192.7 is 2 because when 192.7 is expressed in scientific notation as 1.927×10^2, 2 is the exponent of 10^2. *Sometimes we also include the number* 10 *in the definition of order of magnitude* and say that the order of magnitude of 192.7 is 10^2.)

Familiar Terms and Symbols[2]

- Exponential Notation
- Base, Exponent, Power
- Integer
- Whole Number
- Expanded Form (of decimal numbers)
- Square and Cube (of a number)
- Equivalent Fractions

Suggested Tools and Representations

- Scientific Calculator

Rapid White Board Exchanges

Implementing a RWBE requires that each student be provided with a personal white board, a white board marker, and a means of erasing his or her work. An economic choice for these materials is to place sheets of card stock inside sheet protectors to use as the personal white boards and to cut sheets of felt into small squares to use as erasers.

A RWBE consists of a sequence of 10 to 20 problems on a specific topic or skill that starts out with a relatively simple problem and progressively gets more difficult. The teacher should prepare the problems in a way that allows him or her to reveal them to the class one at a time. A flip chart or PowerPoint presentation can be used, or the teacher can write the problems on the board and either cover some with paper or simply write only one problem on the board at a time.

The teacher reveals, and possibly reads aloud, the first problem in the list and announces, "Go". Students work the problem on their personal white boards as quickly as possible and hold their work up for their teacher to see their answers as soon as they have the answer ready. The teacher gives immediate feedback to each student, pointing and/or making eye contact with the student and responding with an affirmation for correct work such as, "Good job!", "Yes!", or "Correct!", or responding with guidance for incorrect work such as "Look again," "Try again," "Check your work," etc. In the case of the RWBE, it is not recommended that the feedback include the name of the student receiving the feedback.

If many students have struggled to get the answer correct, go through the solution of that problem as a class before moving on to the next problem in the sequence. Fluency in the skill has been established when the class is able to go through each problem in quick succession without pausing to go through the solution of each problem individually. If only one or two students have not been able to successfully complete a problem, it is appropriate to move the class forward to the next problem without further delay; in this case find a time to provide remediation to that student before the next fluency exercise on this skill is given.

[2] These are terms and symbols students have seen previously.

Sprints

Sprints are designed to develop fluency. They should be fun, adrenaline-rich activities that intentionally build energy and excitement. A fast pace is essential. During Sprint administration, teachers assume the role of athletic coaches. A rousing routine fuels students' motivation to do their personal best. Student recognition of increasing success is critical, and so every improvement is acknowledged. (See the Sprint Delivery Script for the suggested means of acknowledging and celebrating student success.)

One Sprint has two parts with closely-related problems on each. Students complete the two parts of the Sprint in quick succession with the goal of improving on the second part, even if only by one more.

Sprints are not to be used for a grade. Thus, there is no need for students to write their names on the Sprints. The low-stakes nature of the exercise means that even students with allowances for extended time can participate. When a particular student finds the experience undesirable, it is recommended that the student be allowed to opt-out and take the Sprint home. In this case, it is ideal if the student has a regular opportunity to express the desire to opt-in.

With practice, the Sprint routine takes about 8 minutes.

Sprint Delivery Script

Gather the following: stopwatch, a copy of Sprint A for each student, a copy of Sprint B for each student, answers for Sprint A and Sprint B. The following delineates a script for delivery of a pair of Sprints.

This sprint covers: *topic.*

Do not look at the Sprint, keep it turned face down on your desk.

There are xx problems on the Sprint. You will have 60 seconds. Do as many as you can. I do not expect any of you to finish.

On your mark, get set, GO.

60 seconds of silence.

STOP. Circle the last problem you completed.

I will read the answers. You say "YES" if your answer matches. Mark the ones you have wrong. Don't try to correct them.

Energetically, rapid-fire call the answers ONLY.

Stop reading answers after there are no more students answering, "Yes."

Fantastic! Count the number you have correct, and write it on the top of the page. This is your personal goal for Sprint B.

Raise your hand if you have 1 or more correct. 2 or more, 3 or more...

Let us all applaud our runner up, [insert name] with x correct. And let us applaud our winner, [insert name], with x correct.

You have a few minutes to finish up the page and get ready for the next Sprint.

Students are allowed to talk and ask for help; let this part last as long as most are working seriously.

Stop working. I will read the answers again so you can check your work. You say "YES" if your answer matches.

Energetically, rapid-fire call the answers ONLY.

Optionally, ask students to stand and lead them in an energy-expanding exercise that also keeps the brain going. Examples are jumping jacks or arm circles, etc. while counting by 15's starting at 15, going up to 150 and back down to 0. You can follow this first exercise with a cool down exercise of a similar nature, such as calf raises with counting by one-sixths $\left(\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}, 1 \ldots\right)$.

Hand out the second Sprint and continue reading the script.

Keep the Sprint face down on your desk.

There are xx problems on the Sprint. You will have 60 seconds. Do as many as you can. I do not expect any of you to finish.

On your mark, get set, GO.

60 seconds of silence.

STOP. Circle the last problem you completed.

I will read the answers. You say "YES" if your answer matches. Mark the ones you have wrong. Don't try to correct them.

Quickly read the answers ONLY.

Count the number you have correct, and write it on the top of the page.

Raise your hand if you have 1 or more correct. 2 or more, 3 or more, ...

Let us all applaud our runner up, [insert name] with x correct. And let us applaud our winner, {insert name], with x correct.

Write the amount by which your score improved at the top of the page.

Raise your hand if you improved your score by 1 or more. 2 or more, 3 or more...

Let us all applaud our runner up for most improved, [insert name]. And let us applaud our winner for most improved, [insert name].

You can take the Sprint home and finish it if you want.

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic A	Constructed response with rubric	8.EE.A.1
End-of-Module Assessment Task	After Topic B	Constructed response with rubric	8.EE.A.3, 8.EE.A.4